Fatigue – a survival kit

Solid Mechanics

FATIGUE

- A Crash Course

Fatigue initiation
Limited High Cycle Fatigue life

Low Cycle Fatigue
Stress Concentrations in LCF

Stress cycle identification
Damage accumulation

Multiaxial fatigue initiation

Crack propagation
Retardation effects

Crack closure
Crack growth arrestment
Crack growth in mixed mode loading
Damage mechanisms

\[ \sigma_{pl} \quad \text{Plastic shakedown limit} \]

\[ \sigma_{el} \quad \text{Elastic shakedown limit} \]

\[ \sigma_Y \quad \text{Yield limit} \]

\[ \sigma_{FL} \quad \text{Fatigue limit} \]
Fatigue design methods

Parameters

- Magnitude of loading
- Complexity of loading
- Damage of material
- Environment
- Possibility of experimental validations
- Experience
- ...

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Anders Ekberg
Philosophies in Design

Safe Life

- “Safe life” means that the component is designed to last a previously determined life
  - Equivalent stress criterion (no cracks initiated)
  - Fracture mechanics analysis (crack does not propagate enough to cause failure in operational life)

Fail Safe

- Cracks are not allowed to grow to failure. Also, (partial) failure of a part of the component through crack development must not endanger safety

Design steps

- A flaw size, that will lead to fracture, is estimated.
- A tolerable flaw size is defined
- An initial flaw size is defined
- The time to grow the crack from initial to tolerable size is calculated
- Based on this, inspection schemes are defined
Fatigue

How?

- **Micro structural changes** which cause nucleation of permanent damage. Persistent slip bands (PSB)
- Creation of **microscopic cracks**
- Growth and coalescence of microscopic flaws to form "dominant" cracks
- Stable propagation of dominant **macro crack(s)**
- Structural instability or complete failure

Where?

- Initiation where $\sigma/s > 1$
- Normally
  - **weak spots**
    - inclusions
    - small cracks
  - **stress concentrations**
    - inclusions
    - corrosion pits
- Fatigue will then grow (propagate) in the direction of $\max \sigma/s$
- Initiation $\neq$ propagation
Fatigue crack initiation and propagation

◊ Small cracks
  - **Shear** driven
  - Interact with *microstructure*
  - Mostly analyzed by *continuum mechanics* approaches

◊ Large cracks
  - **Tension** driven
  - Fairly *insensitive to microstructure*
  - Mostly analyzed by *fracture mechanics* models
Influencing factors

Stress concentrations

◊ Give rise to increased stress levels

Volumetric effects

◊ Increased volume subjected to high load magnitude gives larger probability of a “weak, highly stressed spot”

◊ Large raw material gives reduced fatigue resistance due to manufacturing quality

Environmental effects

◊ Corrosion
  • Corrosion pits which act as stress concentrators.
  • Cracks will always form due to corrosion -> no fatigue limit

◊ Heat will often decrease the fatigue strength and the fatigue limit may diminish at higher temperatures.
Haigh diagram

\[ \sigma_a = \sigma_{FLP} \]

\[ \sigma_m = 0 \]

\[ \sigma_m = \sigma_{FLP} \]

Surface roughness

Size of raw material

Loaded volume

Reduced Haigh diagram

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Haigh diagram II

$SF_a = \frac{AA'}{AP}$  \( \sigma_m = \text{const} \)

$SF_m = \frac{OB'}{OA}$  \( \sigma_a = \text{const} \)

$SF_{am} = \frac{OC'}{OP}$

$K_f \cdot \sigma_a = \text{const}$

$K_f = 1 + q(K_t - 1)$

$K_f \leq K_t$

$(K_t \cdot \sigma_m, K_f \cdot \sigma_a)$

Service stress

Safety factors
**Wöhler (S-N) curve**

- **Valid only for a certain R-ratio**
- **Given stress amplitude**
- **Gives allowable stress amplitude**
- **Gives pertinent fatigue life**
- **No fatigue damage is induced, the component can sustain an infinite number of load cycles**
- **Given service life**
- **“steel”**
- **“aluminum”**

This slope on the Wöhler curve can be described by the equation:

\[
\sigma_a^m \cdot N_f = K
\]

or be approximated as a straight line.

- **The Wöhler diagram can be used to design for finite (and infinite) life**
- **This can be done either for a given service loading or a given service life**

Given stress amplitude

Gives pertinent fatigue life

No fatigue damage is induced, the component can sustain an infinite number of load cycles

Given service life

Valid only for a certain R-ratio

Given service life
Stress cycle identification – rainflow counting

Depict the loading sequence as a function of time.
- start with largest max or smallest min
- use straight lines between min and max

Let “drops” start from every max and min and stop if:
- it starts from max and passes a larger or equal max
- it starts from min and passes a larger or equal min
- it reaches the run of another drop

Identify closed loops by joining drops

1 passes an equally large maximum
2 passes a larger minimum
3 passes a larger maximum
4 reaches the run of drop 2
5 reaches the run of drop 1
6 “falls out”
7 “falls out”
8 reaches the run of drop 6
1 and 6, 2 and 5, 3 and 4; 7 and 8 form closed loops (i.e. stress cycles)
A assume that, during the service life, we have 500 loadings of type 1 (defined by mid-value and magnitude), 1000 loadings of type 2 and 10000 loadings of type 3.

The Palmgren - Miner rule states that failure occurs when

$$\sum_{j=1}^{J} D_j = \sum_{i=1}^{I} \frac{n_i}{N_i} = 1$$

where $D_j$ is the damage of a load cycle, $n_i$ is the number of applied load cycles of type $i$, and $N_i$ is the pertinent fatigue life.

$$\sum_{j=1}^{J} D_j = \sum_{i=1}^{I} \frac{n_i}{N_i} = \frac{500}{10^3} + \frac{10^3}{10^5} + \frac{10^4}{\infty} = 0.51 < 1$$
Equivalent stress criteria

**Sines criterion**

\[
\sigma_{EQS} = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{1,a} - \sigma_{2,a}\right)^2 + \left(\sigma_{2,a} - \sigma_{3,a}\right)^2 + \left(\sigma_{3,a} - \sigma_{1,a}\right)^2 + c_S \sigma_{h,mid} > \sigma_{eS}}
\]

\[
\sigma_{EQS} = \frac{3}{2} \sigma_{ij,a}^d \sigma_{ij,a}^d + c_S \sigma_{h,mid} > \sigma_{eS}
\]

**Crossland criterion**

\[
\sigma_{EQC} = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{1,a} - \sigma_{2,a}\right)^2 + \left(\sigma_{2,a} - \sigma_{3,a}\right)^2 + \left(\sigma_{3,a} - \sigma_{1,a}\right)^2 + c_C \sigma_{h,max} > \sigma_{eC}}
\]

\[
\sigma_{EQS} = \frac{3}{2} \sigma_{ij,a}^d \sigma_{ij,a}^d + c_C \sigma_{h,max} > \sigma_{eC}
\]

**Dang Van criterion**

\[
\sigma_{EQDV} = \frac{\sigma_{1,a} - \sigma_{3,a}}{2} + c_{DV} \sigma_{h,max} > \sigma_{eDV}
\]

*Valid only for proportional loading (in-phase and fixed principal directions)*
Equivalent stress criteria – components

Shear stress measures

◊ The shear stress initiates microscopic cracks (stage I crack growth)

◊ A static shear stress have no influence on fatigue damage ⇒ the shear stress "amplitude" is employed

Hydrostatic stress

◊ Mean value of normal stresses that opens up cracks (Stage II crack growth)

\[ \sigma_h = \frac{1}{3} \sigma_{ii} = \frac{1}{3} \left( \sigma_{11} + \sigma_{22} + \sigma_{33} \right) \]

regardless of coordinate system (stress invariant)
The deviatoric stress tensor

The stress tensor can be split into deviatoric and volumetric part

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yz} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix} = \begin{bmatrix}
\sigma_{xx} - \sigma_h & \tau_{xy} & \tau_{xz} \\
\tau_{yz} & \sigma_{yy} - \sigma_h & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz} - \sigma_h
\end{bmatrix} + \sigma_h \begin{bmatrix}1 & 0 & 0 \\0 & 1 & 0 \\0 & 0 & 1\end{bmatrix}
\]

\[
= \sigma^d + \sigma_h I = \sigma_{ij}^d + \frac{1}{3} \delta_{ij} \sigma_{kk}
\]

The volumetric part contains the hydrostatic stress
The deviatoric part reflects influence of shear stresses

Midvalue: \( \sigma_{ij,m}^d = \sigma_m^d = \begin{bmatrix}
\sigma_{xx,m}^d & \tau_{xy,m}^d & \tau_{xz,m}^d \\
\tau_{yx,m}^d & \sigma_{yy,m}^d & \tau_{yz,m}^d \\
\tau_{zx,m}^d & \tau_{zy,m}^d & \sigma_{zz,m}^d
\end{bmatrix} \)
(proportional loading)

Amplitude: \( \sigma_{ij,a}(t) = \sigma_{ij}^d(t) - \sigma_{ij,m}^d \) (or \( \sigma_a^d(t) = \sigma^d(t) - \sigma_m^d \) )
Low Cycle Fatigue

- Stresses close to (or at) the **yield** limit
  Small stress increment $\Rightarrow$ large strain increment. Best “resolution” if strains are employed in fatigue model

- Induced fatigue damage due to global plasticity

- Loading above yield limit, (LCF) gives $\Delta\sigma \neq \Delta\varepsilon$
  With stress concentration factor
  $$K_\sigma \equiv \frac{\sigma_{\text{max}}}{\sigma_\infty}$$
  and strain concentration factor
  $$K_\varepsilon \equiv \frac{\varepsilon_{\text{max}}}{\varepsilon_\infty}$$
  we get $K_\varepsilon \neq K_\sigma$
Stress concentrations in LCF – Neuber’s rule

◊ At a stress concentration, Neuber’s rule gives the relation between stress and strain as

\[
\sigma_{\text{max}} \cdot \varepsilon_{\text{max}} = \frac{K_f^2 \sigma_{\infty}^2}{E}
\]

\[
\sigma_{\text{max, a}} \cdot \varepsilon_{\text{max, a}} = \frac{K_f^2 \sigma_{\infty, a}^2}{E}
\]

◊ This equation has two unknowns.

◊ Stress and strain must also fulfill constitutive relationship (for cyclic loading)

⇒ 2 equations and 2 unknowns
LCF Design Rules

According to Morrow, the relationship between strain amplitude, $\varepsilon_a$, and pertinent number of load cycles to failure, $N_f$ can be written as

$$\varepsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

or, with a static mean stress $\sigma_m \neq 0$

$$\varepsilon_a = \frac{(\sigma'_f - \sigma_m)}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

According to Coffin – Manson, the relationship can be simplified as

$$\varepsilon_a = 1.75 \frac{\sigma_{UTS}}{E} N_f^{-0.12} + 0.5 D^{0.6} N_f^{-0.6}$$
Fatigue crack growth

- In experiments, crack propagation has been measured as a function of the stress intensity factor.

- There exists a threshold value of $\Delta K$ below which fatigue cracks will not propagate.

- At the other extreme, $K_{\text{max}}$ will approach the fracture toughness $K_C$, and the material will fail.

- Linear relationship between $\log\left(\frac{da}{dN}\right)$ and $\Delta K$ in region II.

- $da/dN$ depends also on crack size. This is not shown in the plot.
Paris’ law

- Paris’ law can be written as
  \[ \frac{da}{dN} = C\Delta K^m \]
  \((C \text{ and } m \text{ are material parameters})\)

1. Find stress intensity factor for the current geometry
2. Find crack length corresponding to \(K_{\text{max}} = K_C\)
3. (Check if LEFM is OK)
4. Integrate Paris’ law
5. Solve for the number of stress cycles corresponding to failure

- Paris’ law does not account for
  - mean stress effects (described by the R-ratio)
  - history effects (introduced by \(H\))

and is only valid for
  - uniaxial loading
  - “long cracks”
  - LEFM-conditions
Variable amplitude loading

- A (tensile) overload will introduce (compressive) residual stresses.
- These residual stresses will influence $\Delta K$ and thus the rate of crack propagation.
- The Wheeler model is used to define the reduction of the crack growth rate due to overload.

The reduction factor is defined as

$$\Phi_R = \left( \frac{\Delta a + d_c}{d_0} \right)^\gamma$$

Reduced crack growth rate is then calculated as

$$\left( \frac{da}{dN} \right)_R = \Phi_R \frac{da}{dN}$$
Crack closure

Normally cracks only grow when they are open.

The Elber accounts for crack closure, also for tensile loads by defining an effective stress intensity range:

◊ Paris law
  \[ \Delta K \equiv K_{\text{max}} - K_{\text{min}} \]
  \[ K_{\text{min}} = \max[K_{\text{min}}, 0] \]

◊ Elber correction for crack closure at \( K = K_{\text{op}} \)
  \[ \Delta K_{\text{eff}} \equiv K_{\text{max}} - K_{\text{op}} \]

◊ Modified Paris law
  \[ \frac{da}{dN} = C \Delta K_{\text{eff}}^m \]

◊ Empirical relation
  \[ K_{\text{op}} = \varphi(R)K_{\text{max}}, \text{ where} \]
  \[ \varphi(R) = 0.25 + 0.5R + 0.25R^2 \quad -1 \leq R \leq 1 \]
Crack closure

The only difference when using Elber correction is in a new, higher $K_{\text{min}}$.

Using Elber correction in Paris law is non-conservative (predicts a longer fatigue life) compared to “standard” Paris’ law.
Crack arrest at different scales

A. The load magnitude is below the fatigue limit \( \Rightarrow \) we will not initiate any (macroscopic cracks)

B. The applied load gives a stress intensity below the fatigue threshold \( \Rightarrow \) macroscopic cracks will not continue to grow

\[
K_{I,\text{th}} = \alpha U \sigma \sqrt{\pi a}
\]

\[\log \Delta \sigma - \log \Delta \sigma_e\]

No fatigue failure

No propagation

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