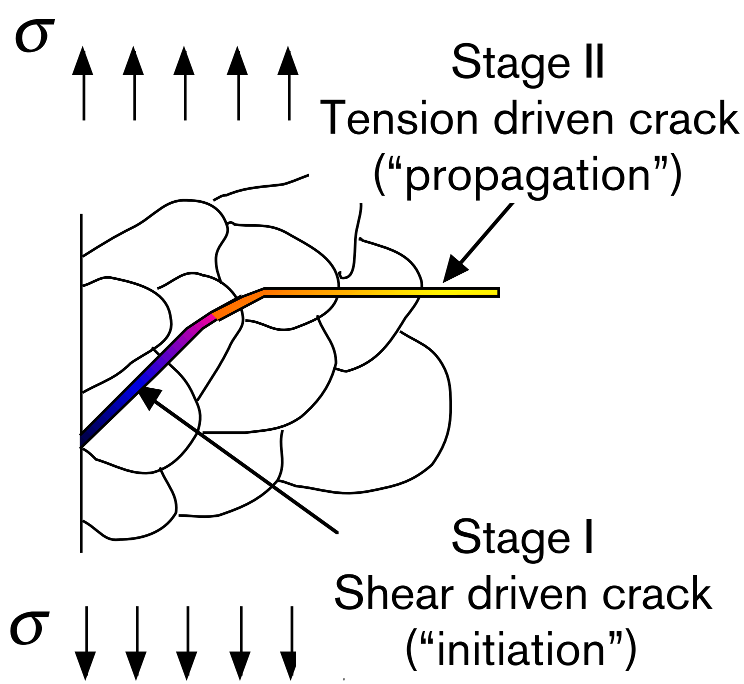
# Fatigue crack propagation

## Repetition – Crack initiation and growth

- Small cracks
  - **Shear driven**
  - Interact with **microstructure**
  - Mostly analyzed by **continuum mechanics** approaches
- Large cracks
  - Tension driven
  - Fairly **insensitive to microstructure**
  - Mostly analyzed by **fracture mechanics** models

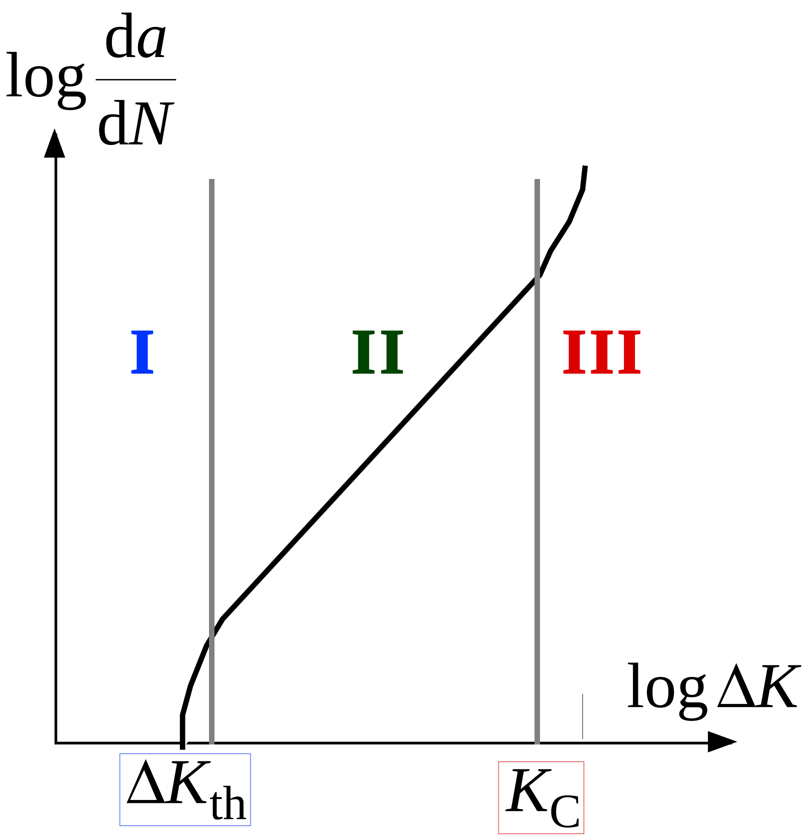


# Stress intensity factors and fracture

- In **static loading**, the stress intensity factor for a small crack in a large specimen can be expressed as $K_{\mathrm{I}} = f\left(\sigma, \sqrt{a}\right)$ where $f$ depends on geometry
  - If the stress is kept constant, we will get **fracture for** a certain crack length, $a = a_{\mathrm{C}}$, which will give $K_{\mathrm{I}} = K_{\mathrm{IC}}$.
  - For $a < a_{\mathrm{C}}$($K_{\mathrm{I}} < K_{\mathrm{IC}}$) the crack will **not propagate** (in theory)
- In **dynamic loading**, we will still get **fracture if** the stress intensity factor, for some instant of time, exceeds $K_{\mathrm{I}} = K_{\mathrm{IC}}$
  - However, for $K_{\mathrm{I}} < K_{\mathrm{IC}}$, the crack may still propagate. Since this means that $a$ (and $K_{\mathrm{I}}$) will increase, we will **eventually obtain fracture** when $a = a_{\mathrm{C}}$.

# Crack growth as a function of ΔK

- In experiments, crack propagation has been measured as a function of the stress intensity factor

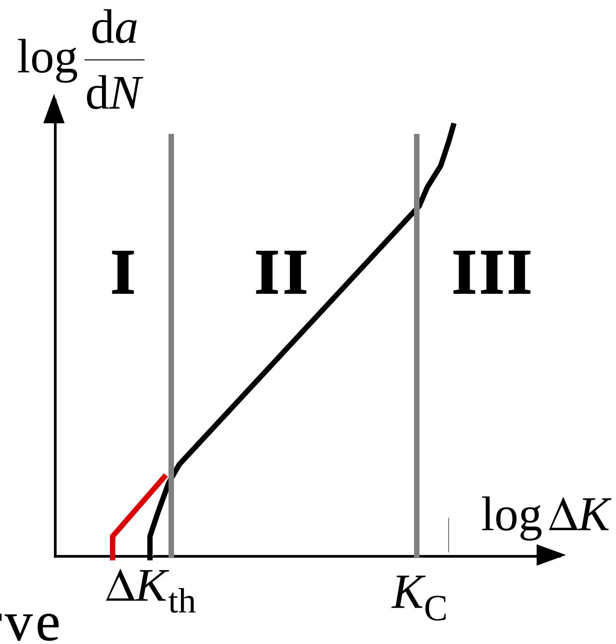



- There exists a **threshold** value of ΔK below which **fatigue cracks will not propagate**
- At the other extreme, $K_{max}$ will approach the fracture toughness $K_C$, and **the material will fail**
- A l**inear relationship** between $\log\left({}^{da}\!/_{dN}\right)$ and $\Delta K$ in **region II**
- Note that ΔK depends on the crack size. This is not shown in the plot

# Crack growth in region I

- For small $\Delta K$ (region I), crack propagation is **difficult to predict** since it depends on **microstructure** and **flow properties** of the material
- Here, the growth may even come to an **arrest**
- Crack growth rate is **sensitive to the size of the grains**. Finer grains gives
  - Closer spacing of grain boundaries, which the crack has to break through
  - Increased yield stress (normally)
  - Decreased roughness of the crack
- Crack growth predicted by
  - models of type $\mathrm{d}a/\mathrm{d}N = f\left(\Delta\gamma_\mathrm{p}\right)$,

    where $\Delta\gamma_\mathrm{p}$ is plastic shear strain range
  - empirical adjustment of $\Delta \mathrm{K}$ - $\mathrm{d}a/\mathrm{d}N$–curve



# Crack growth in region II and III

## Region II

- For larger magnitudes of $\Delta K$ (**region II**), the crack growth rate will be governed by a **power law** (such as Paris' law)
- The crack growth rate is fairly **insensitive to the microstructure** (however, the constants $m$ and $C$ are, of course, different for different materials)
- If region II includes the dominating part of the fatigue life, the fatigue life can be directly estimated by **integrating Paris' law**

## Region III

- If the stress intensity ratio is increased even further (**region III**), the crack growth rate will accelerate and finally fracture will occur
- The behavior of this fracture is rather **sensitive to the microstructure and flow properties** of the material.

# Crack propagation laws – introduction

It has been found that, for dynamic loading of a crack, the **three most important factors** determining the propagation (growth) of the crack are

$\Delta K \equiv K_{\text{max}} - K_{\text{min}}$ – the **stress intensity range**
$R \equiv K_{\text{min}} / K_{\text{max}}$ – the **stress intensity ratio**
$\mathcal{H}$ – the **stress history**

◇ Thus, the **crack growth rate** (i.e. growth per stress cycle) can be expressed as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = f(\Delta K, R, \mathcal{H})$$

where ${\mathrm{d}a}/{\mathrm{d}N}$ is the crack growth per stress cycle

# Paris' law

- Paris' law can be written as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^m$$

where $C$ and $m$ are material parameters

- One of the first (1962) and most widely used fatigue crack propagation criteria

**"Algorithm"**

1. Find stress intensity factor for the current geometry
2. Find crack length corresponding to $K_{\text{max}} = K_{\text{C}}$
3. Check if the requirements for linear elastic fracture mechanics are fulfilled
4. Integrate Paris' law
5. Solve for the number of stress cycles corresponding to failure

**Important**

- If the stress intensity factor includes a geometric function of $a$, estimated (or analytic) values of this function has to be used

# Paris' law – drawbacks

- Compared to a "general" crack propagation criterion

$$\frac{\mathrm{d}a}{\mathrm{d}N} = f(\Delta K, R, \mathcal{H})$$

Paris' law does not account for

  - **mean stress effects** (described by the $R$-ratio)
  - **history effects** (introduced by $\mathcal{H}$)

- Further, Paris law is only valid in conditions with
  - **uniaxial loading**
  - **"long cracks"**
  - LEFM-conditions

- We will have a closer look at
  - **short crack theory**
  - **retardation models** due to overloads
  - **crack closure** effects
  - crack propagation in **multiaxial loading**

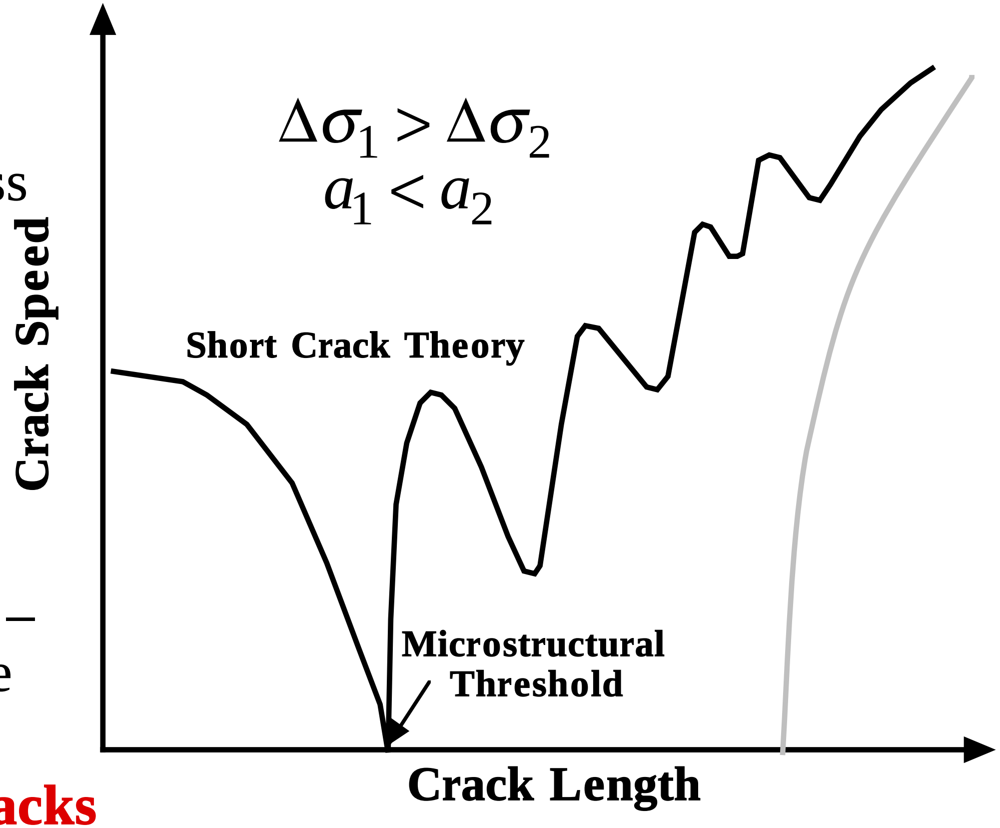
# Short cracks

- So far

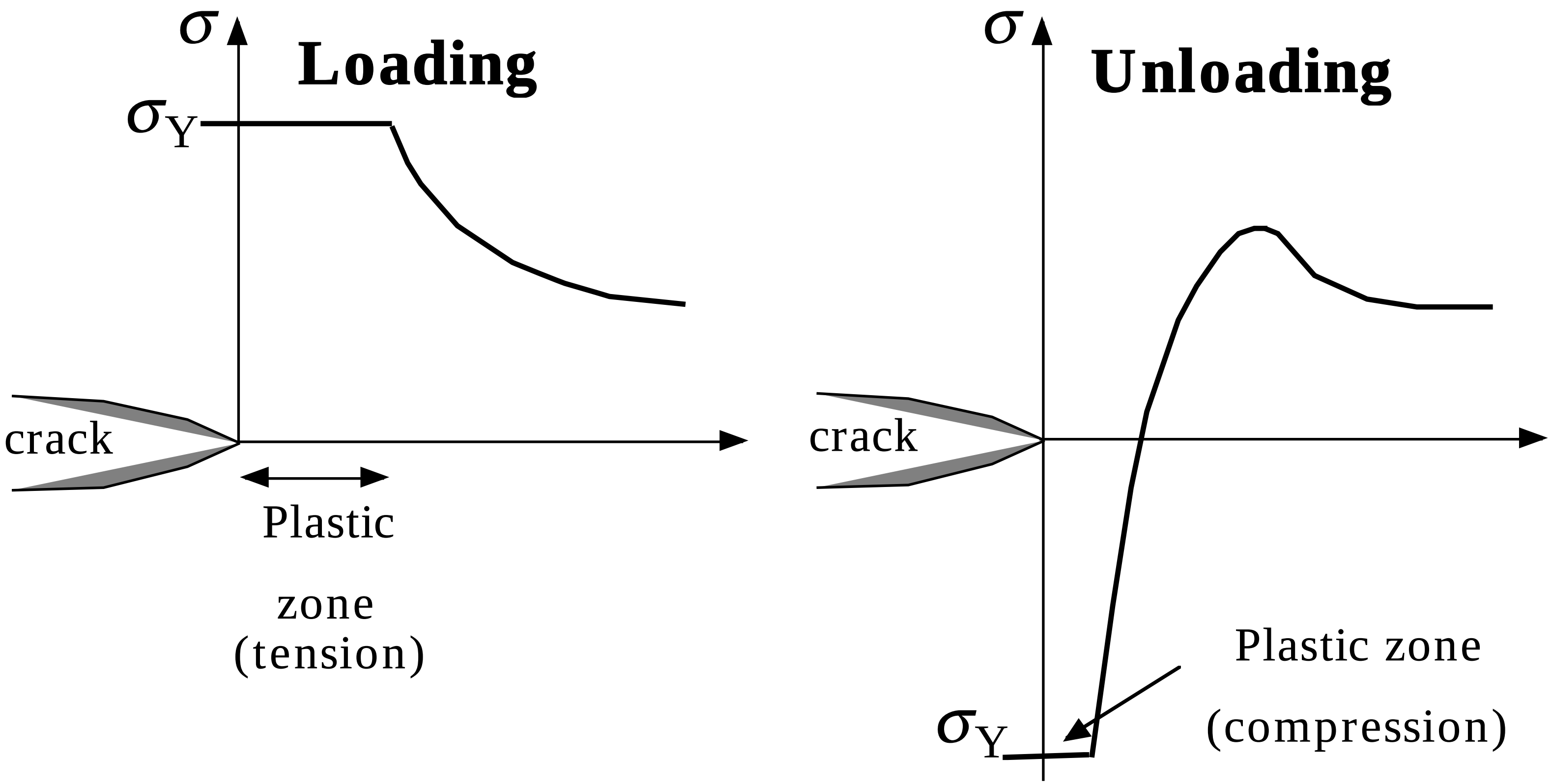
$$\frac{\mathrm{d}a}{\mathrm{d}N} = f(\Delta K)$$

where $\Delta K$ depends on the amplitude of the normal stress (and geometry)

- But short cracks are shear stress driven also LEFM is not valid
- Two types of short cracks
  - **mechanically short cracks** – propagate faster than large cracks with same $\Delta K$
  - **microstructurally short cracks** – interact closely with the microstructure and grow fast



# Variable amplitude loading ($\mathcal{H}$)

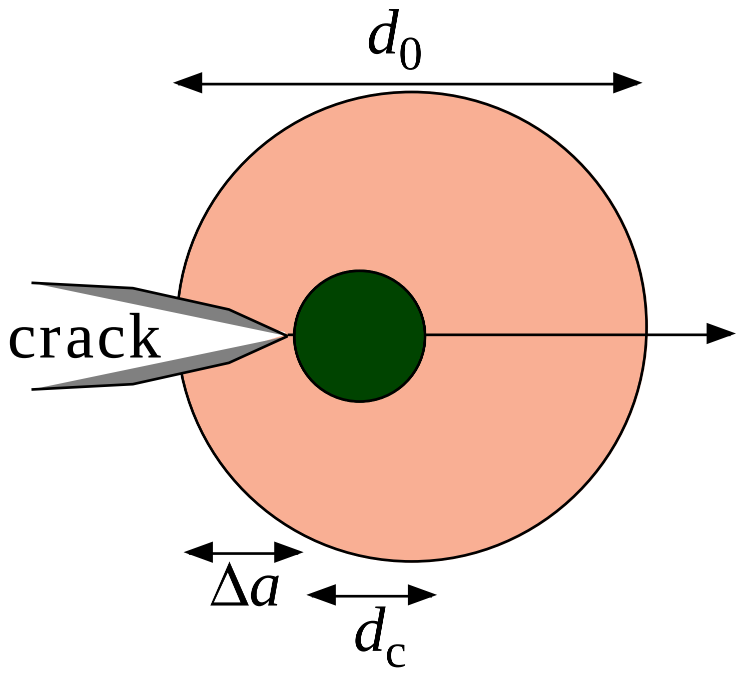



- A (tensile) overload will introduce (compressive) **residual stresses**
- These residual stresses will influence $\Delta K$ and thus the **rate of crack propagation**

# The Wheeler model

- The Wheeler model is used to define the reduction of the crack growth rate due to an overload
- The **reduction factor** is defined as

$$\Phi_R = \left( \frac{\Delta a + d_c}{d_0} \right)^{\gamma}$$




- The **reduced crack growth rate** is then calculated as

$$\left( \frac{da}{dN} \right)_R = \Phi_R \frac{da}{dN}$$

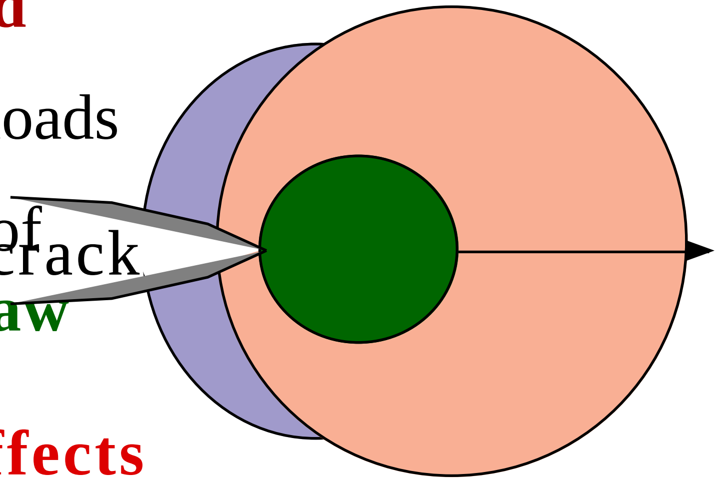
# Variable Amplitude Loading, cont'd

- The Wheeler model is appropriate for **single overloads**

- The reduction of crack growth rate acts only as long as the cracks "**current plastic zone**" is **within** the **plastic zone from the overload**

- Multiple overloads or "stochastic" loads

  - **Cycle-by-cycle integration** of
  - **Appropriate crack growth law** that takes
  - **Retardation/acceleration effects** into account




- "Normal" crack propagation laws are **usually conservative**

# Crack closure ($R$)

Elber, in 1970, discovered that crack closure exists in cyclic loading, **even for loads that are greater than zero**

This **crack closure will decrease the fatigue crack growth rate** by reducing the effective stress intensity range

- The stress intensity rate

$$\Delta K \equiv K_{\max} - K_{\min}$$

$$K_{\min} = \max[K_{\min}, 0]$$

- Crack closure att $K=K_{op}$ gives

$$\Delta K_{\text{eff}} \equiv K_{\max} - K_{op}$$

- Paris law using effective stress intensity rate

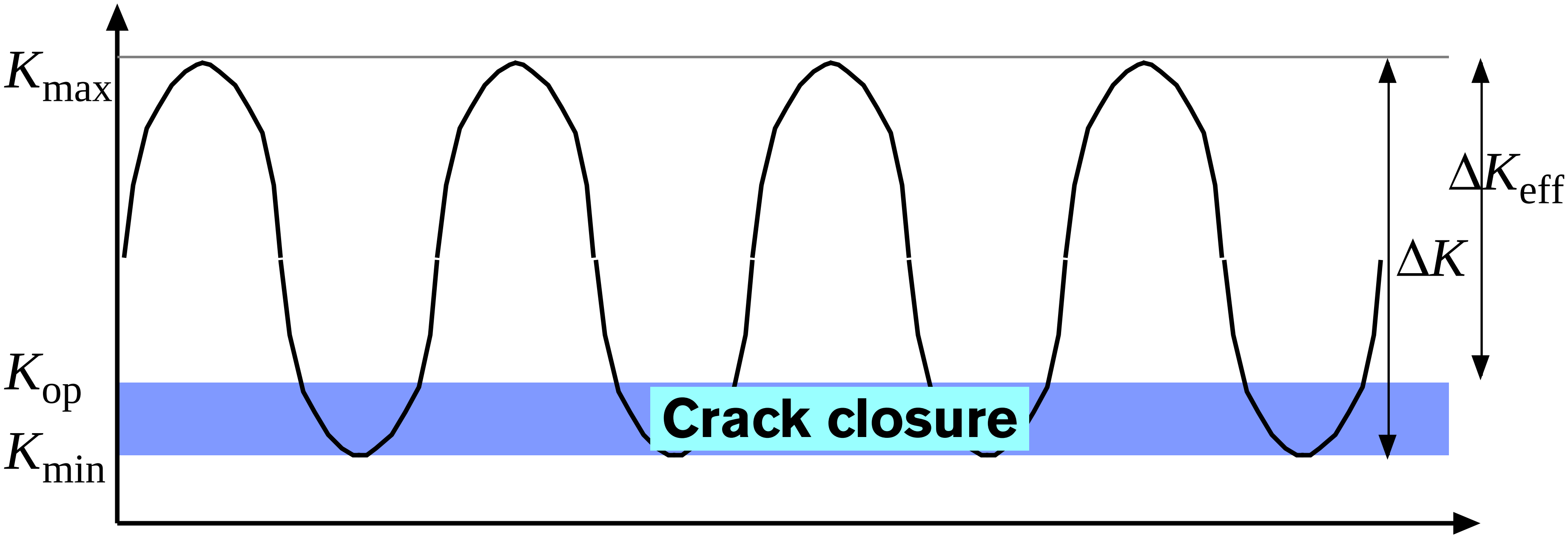
$$\mathrm{d}a/\mathrm{d}N = C\Delta K_{\text{eff}}^{m}$$

- Empirical relation

$$K_{op} = \varphi(R) K_{\max}$$

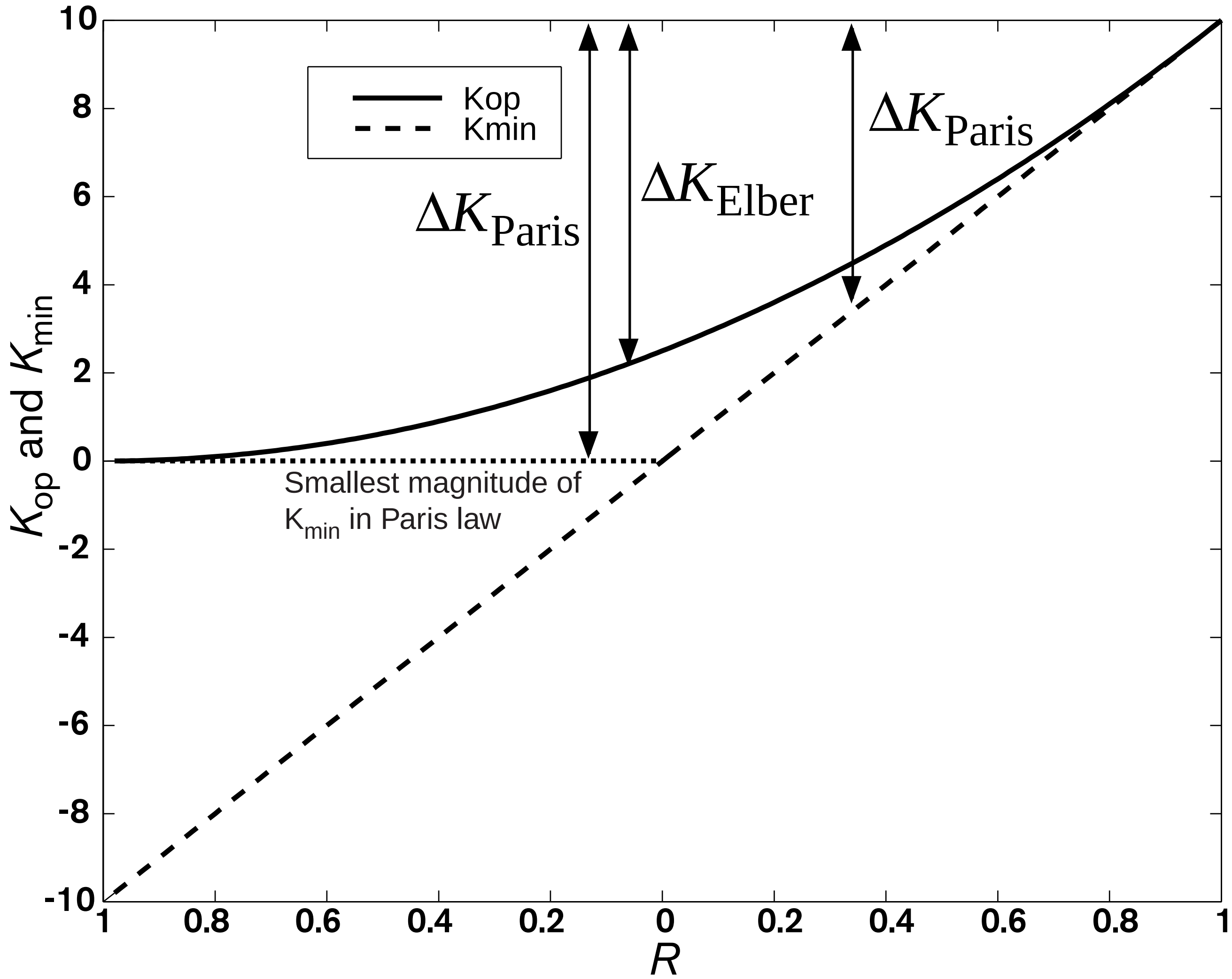
$$\varphi(R) = 0.25 + 0.5R + 0.25R^2 \quad -1 \leq R \leq 1$$

# Crack closure and arrestment




- If the crack is **closed throughout the stress cycle**, **the crack will arrest**
- This is **not the only mechanism** of a crack to arrest!

# Crack closure and arrestment – II




The only difference when using Elber correction is in **a new, higher** $K_{\mathrm{min}}$

Using Elber correction in Paris law is **conservative** (predicts a longer fatigue life)

# Crack arrestment

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K_{\mathrm{eff}}^{m} = C\left(\frac{\Delta K_{\mathrm{eff}}}{\Delta K}\Delta K\right)^{m}$$

$$= C\left(\frac{K_{\max} - K_{\mathrm{op}}}{\Delta K}\Delta K\right)^{m}$$

$$= C\left(\left(\frac{K_{\max}}{K_{\max} - K_{\min}} - \frac{K_{\mathrm{op}}}{\Delta K}\right)\Delta K\right)^{m}$$

$$= C\left(\left(\frac{1}{\frac{K_{\max}}{K_{\max}} - \frac{K_{\min}}{K_{\max}}} - \frac{K_{\mathrm{op}}}{\Delta K}\right)\Delta K\right)^{m}$$

$$= C\left(\left(\frac{1}{1-R} - \frac{K_{\mathrm{op}}}{\Delta K}\right)\Delta K\right)^{m}$$

- For

$$\frac{1}{1-R} - \frac{K_{\mathrm{op}}}{\Delta K} = 0$$

we get

$$\Delta K = K_{\mathrm{op}}(1-R) = \Delta K_{\mathrm{th}}$$

and

$$\frac{\mathrm{d}a}{\mathrm{d}N} = 0$$

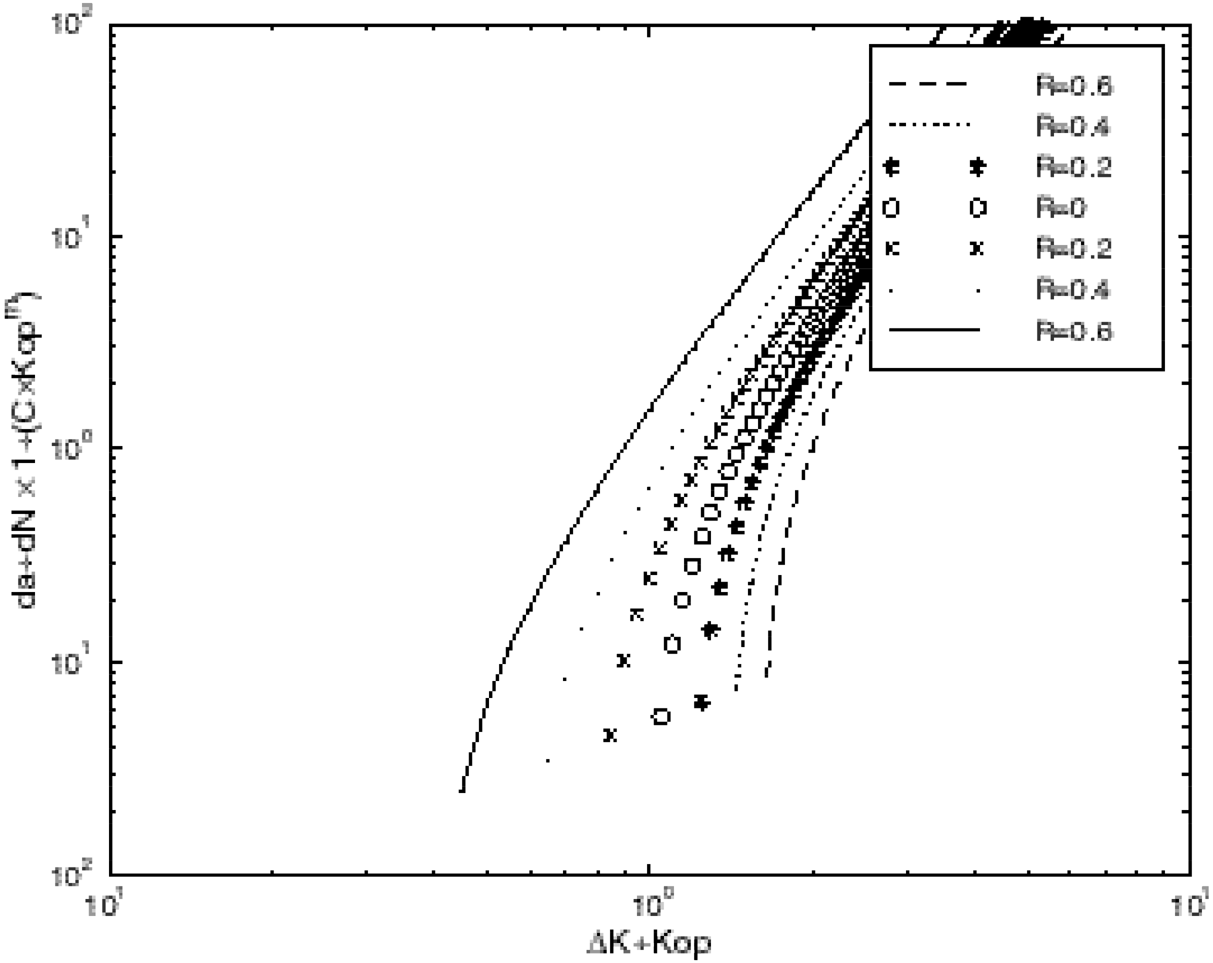
- For

$$\frac{1}{1-R} - \frac{K_{\mathrm{op}}}{\Delta K} = 1$$

we get

$$\Delta K = K_{\mathrm{op}}\left(\frac{1}{R} - 1\right)$$

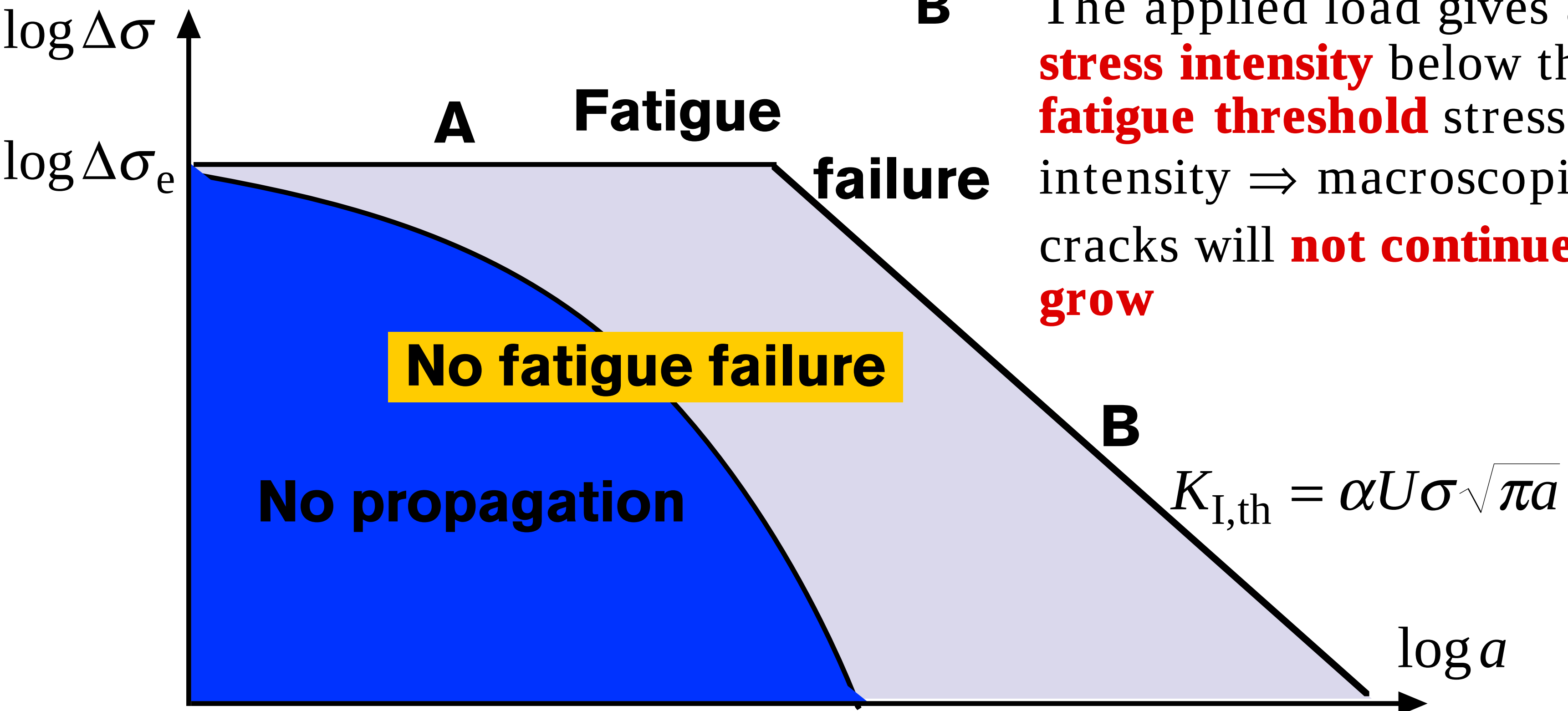
# Crack growth treshold




By taking crack closure into account (using Elber correction), we can model a *R*-ratio dependence

- **compressive** mid stress ⇒ **slower** crack propagation
- **tensile** mid stress ⇒ **faster** crack propagation

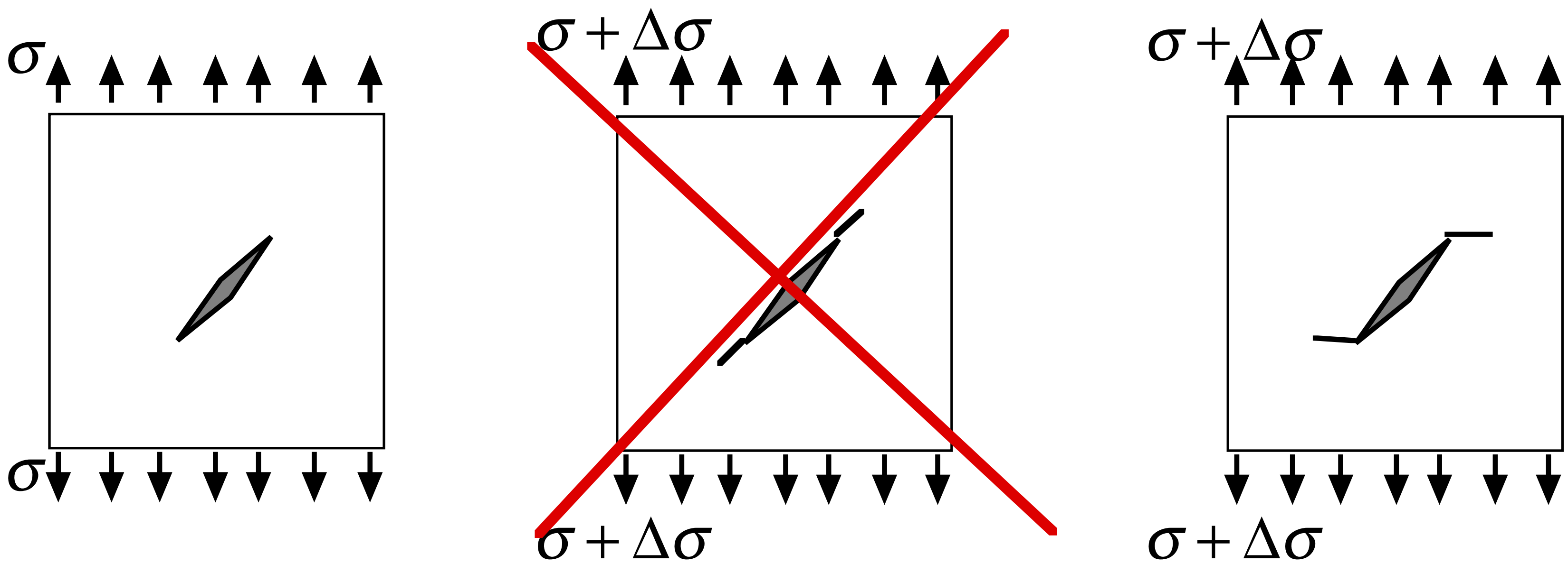
# Crack arrest at different scales




**A** The load magnitude is **below the fatigue limit** $\Rightarrow$ we will **not initiate** any (macroscopic cracks)

**B** The applied load gives a **stress intensity** below the **fatigue threshold** stress intensity $\Rightarrow$ macroscopic cracks will **not continue to grow**

# Cracks in mixed mode loading




- Cracks that are loaded in mixed mode, will normally tend to **propagate in pure mode I**

- One exception is when a crack propagates along a weak zone (*e.g.* a weld). In this case, an effective stress intensity factor can be employed

$$\Delta K_{\text{eff}} = \sqrt{\Delta K_{\text{I}}^2 + (0.8 \cdot \Delta K_{\text{II}})^2}$$

# Crack propagation – summary

- Under **one dimensional, elastic conditions** and **constant load range** **Paris' law**, can predict fatigue life of large cracks
- Under **variable amplitude** loading, plastic residual stress fields mostly gives a **decrease** in crack growth rate.
- **Microstructurally small cracks** interact closely with microstructure. **Mechanically small cracks** propagate faster than long cracks.
- **Closure effects** of large cracks can give a pronounced effect. It's one mechanism behind **crack arrestment**
- In **multiaxial loading**, most cracks tend to propagate in **pure mode I**

## Less "mature" areas

- Cases **where LEFM is not applicable**
- The propagation of **short**, especially microstructurally short, cracks
- Cases where **crack closure** and **crack friction** has a profound effect
- Conditions of **variable amplitude loading**
- **Multiaxial** loading conditions