Fatigue - Calculations

This paper may be used at examination

1 High Cycle Fatigue

1.1 Design for infinite fatigue life using the Haigh diagram

- Determine
  - fatigue limit in alternating tension-compression $\sigma_{FL}$
  - fatigue limit in pulsating tension $\sigma_{FLP}$
  - ultimate tensile stress $\sigma_{UTS}$
  - the yield limit $\sigma_y$

If these are not known, use rules of thumb on p.6 in "Material Fatigue"

- Plot these points and joining lines in a diagram of stress amplitude vs. mid stress during a stress cycle, according to fig. 3 on p. 7 in "Material Fatigue"

- If feasible, reduce the Haigh diagram on the amplitude axis with respect to
  - surface roughness, $\kappa$
  - stressed volume (not in cases of stress concentrations), $\delta$
  - volume of raw material, $\lambda$

The reduction factors are given in diagrams on pp. 9-10 in "Material Fatigue". See also fig. 8 on p. 11 in "Material Fatigue"

- Insert the operational stress, i.e. the point showing the stress amplitude and mid stress at the operational loads in the studied material point

- In the case of stress concentrations, increase the operational stress with
  - $K'_t$ on the mid stress axis
  - $K'_f$ on the amplitude axis

$K'_t$ is obtained from diagrams on pp. 63-67 in "Material fatigue"

$K'_f$ is obtained from $K'_t$ using eq.3 and fig. 9 on p.12 in "Material fatigue"

- Determine the feasible safety factors (dependent on the type of loading) according to eq. 4a,b,c on p. 14 in "Material fatigue"

1.2 Design for finite fatigue life using the Wöhler curve

- Determine the fatigue life corresponding to two different stress amplitudes
- Determine the equation of the Wöhler curve in one of the following ways
  a) by solving for the constants $m$ and $K'$ in eq. 3 on p. 4 in "Material fatigue"
  b) by assuming the Wöhler curve to be a straight line in a diagram plotting the stress amplitude vs. the logarithm of the pertinent fatigue life: 
- Introduce the two amplitudes and their pertinent fatigue lives as two points in the diagram
- Determine the slope of the joining line
- Use the slope to express the equation of the straight line forming the Wöhler curve
- For a given stress amplitude, solve for pertinent fatigue life by using the derived equation of the Wöhler curve
- For a given fatigue life, solve for allowable stress amplitude by using the derived equation of the Wöhler curve

1.3 Designing for infinite fatigue life using equivalent stress approaches

- Derive material parameters from uniaxial fatigue limits by introducing these loads into the equivalent stress criteria (given in the first versions of eqs. 16, 17 and 18 in “Fatigue – a survey”). This is shown in example on pp.56-57 in “Fatigue – a survey”
- If no shear stresses are included in the stress tensor,
  - Derive equivalent stresses from the first versions of eqs. 16, 17 and 18 on p.24 in “Fatigue – a survey”
  - Derive safety factors of fatigue initiation according to eq. 19 on p.26 in “Fatigue – a survey”
- If the stress tensor includes shear stresses, the third versions of eqs. 16, 17 can be used for the Sines and Crossland criteria in order to avoid calculating the principal stresses. In these cases:
  - Calculate the (time dependent) hydrostatic stress (footnote 6 on p.20 in “Fatigue – a survey”)
  - Derive the (time dependent) deviatoric stress tensor according to eq. 4 in “Fatigue – a survey” (see also p.54)
  - Derive the mid value of the deviatoric stress tensor, see example on p.22
  - Derive the time dependent “amplitude” of the deviatoric stress tensor according to eq. 11 on p.23 in “Fatigue – a survey”
    - Derive \( \sqrt[3]{ \frac{1}{2} \sum_{ij} \sigma_{ij}^d(t) } \) and find the max value (corresponding to max value of the time dependent function)
  - Derive the equivalent stress according to the third versions of eqs. 16, 17 on p.24 in “Fatigue – a survey”
  - Derive safety factors of fatigue initiation according to eq. 19 on p.26 in “Fatigue – a survey”
- If the stress tensor includes shear stresses, principal stresses have to be derived in order to employ the Dang Van criterion:
  - Derive (the time dependent) principal values of the stress matrix using the equation under point 5 in appendix 1 on p. 72 in “Fatigue – a survey”
  - Employ the amplitude of the derived principal stresses in the first version of eq. 18 on p.24 in “Fatigue – a survey”
- Derive safety factor of fatigue initiation according to eq. 19 on p.26 in “Fatigue – a survey”

2 Low Cycle Fatigue

2.1 Stresses and strains at notches according to Neuber
- Determine the nominal stress of the component
- Determine the constitutive relation of the material (for instance a Ramberg-Osgood relationship as given in eq. 2 on p. 34 in “Material fatigue”)
- Use the Neuber hyperbola, eq. 8c,d on p. 49 in “Material fatigue”
- The constitutive relationship and the Neuber hyperbola define a system of two equations and two unknowns ($\varepsilon_{a,\text{max}}$ and $\sigma_{a,\text{max}}$ at the notch). Solve for these unknowns

2.2 Fatigue life according to Coffin and Manson
- The fatigue life is given by eq. 5g,h on p.42 in “Material fatigue” for a given strain amplitude (for instance derived using the Neuber hyperbola). Normally, the equation has to be solved numerically
- For a given fatigue life, the same equations will give the allowable strain amplitude

2.3 Fatigue life according to Morrow
- The fatigue life is given by eq. 4a on p.40 in “Material fatigue” for a given strain amplitude (solved numerically). For a given fatigue life, the same equations will give the allowable strain amplitude
- If the mid stress during a strain (stress) cycle is not zero, this can be compensated for by using the modified criterion eq. 4b on p. 41 in “Material fatigue”

3 Fatigue Crack Propagation

3.1 Paris’ law of crack propagation
- The crack growth rate is given by eq. 27 on p. 33 in “Fatigue – a survey”
- By integrating this relationship, the fatigue life can be found. Example:
  - An initial crack size is given as $a_{\text{ini}}$
  - Final crack size (corresponding to fracture) is given $a_c$ that gives a stress intensity factor equal to $K_c$
  - The stress intensity range is given as $\Delta K = (K_{\text{max}} - \max(K_{\text{min}}, 0))$ (see p. 33 in “Fatigue – a survey”)
  - Integrate Paris’ law from $a_{\text{ini}}$ to $a_c$ and the fatigue life from 0 to $N_f$ stress cycles, where $N_f$ is the fatigue life to fracture
- If the expression for the stress intensity factor contains a geometric variable that varies with \(a\), approximate with a (conservative) constant value

3.2 **Elber correction for crack closure**
- Same as Paris’ law, but the stress intensity range is given as
  \[ \Delta K_{\text{eff}} = K_{\text{max}} - \max[K_{\text{min}}, K_{\text{op}}] \] (see p. 35 in “Fatigue – a survey”)
- \(K_{\text{op}}\) is given by eq. 30 and 31, p.35 in “Fatigue – a survey”

3.3 **Wheeler correction for retardation due to overload**
- The retarded crack growth rate is given by eq. 35, 36 and fig. 9, pp. 38-39 in “Fatigue – a survey”

3.4 **Fail safe design**
- Design steps are given in sec. 7.1, p. 50 in “Fatigue – a survey”

4 **Miscellaneous**

4.1 **Damage accumulation according to Palmgren-Miner**
- Palmgren-Miner’s rule is given in eq. 10 p. 24 in “Material fatigue”
- Example is given on pp. 24-29 and pp. 54-55 in “Material fatigue”

4.2 **Identification of stress (or strain or load) cycles**
- The rainflow counting algorithm is described and exemplified on pp. 27-29 in “Material fatigue”