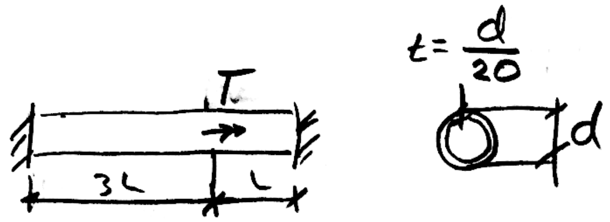
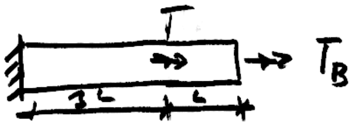
1



Bestäm τ_{max} uttryckt i T och d!

a) Inför statiskt övertalig:



$$\varphi_B = \frac{T \cdot 3L}{GK} + \frac{T_B \cdot 4L}{GK} = 0$$

$$T_B = \frac{GK}{4L} \cdot \left(-\frac{T \cdot 3L}{GK}\right) = -\frac{3}{4}T$$

Snitt moment:

- höger om T: $M_v = T_B = -\frac{3}{4}T$
- vänster om T: $M_v = T + T_B = \frac{1}{4}T$

$$|M_v|_{max} = -\frac{3}{4}T$$

$$\tau_{max} = \frac{|M_v|}{K} \cdot r = \frac{3}{4}\frac{T}{K} \cdot \frac{d}{2}$$

För tunnväggigt rör är $K = \frac{\pi t d^3}{4}$

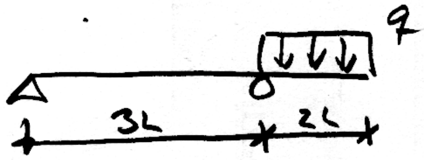
$$\therefore \tau_{max} = \frac{3}{4}\frac{T \cdot 4d}{\pi t d^3 \cdot 2} = \frac{3T}{2\pi t d^2} = \underline{\underline{\frac{30T}{\pi d^3}}} \quad \left[\frac{Nm}{m^3} = Pa\right]$$

b) För solitt tvärsnitt är $K = \frac{\pi d^4}{32}$

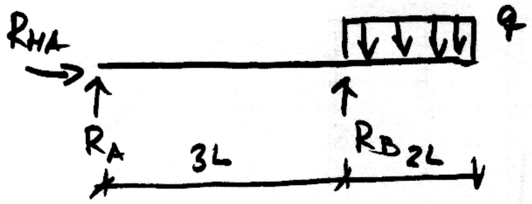
$$\tau_{max} = \frac{3T \cdot d \cdot 32}{8 \cdot \pi d^4} = \frac{12T}{\pi d^3}$$

$$\frac{\tau^b_{max}}{\tau^a_{max}} = \frac{12T \, \pi d^3}{\pi d^3 \, 30T} = \frac{12}{30} = \frac{2}{5} = \underline{\underline{40\%}}$$

2



Statiskt bestämt, frilägg!



$R_{HA} = 0$

$\overset{\curvearrowright}{A}$: $-R_B \cdot 3L + q \cdot 2L \cdot 4L = 0$

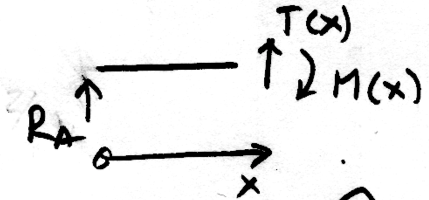
$$R_B = \frac{8L^2}{3L} q = \frac{8}{3} qL$$

$\uparrow$: $R_A + R_B - q \cdot 2L = 0$

$$R_A = 2qL - \frac{8}{3} qL = -\frac{2}{3} qL$$

Snitta

- Till venster om B:



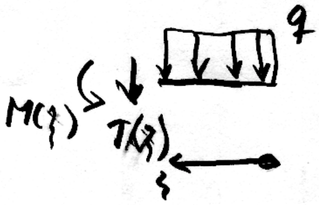
$\uparrow$: $T(x) + R_A = 0$

$$T(x) = -R_A = \frac{2}{3} qL$$

$\overset{\curvearrowright}{x}$: $M(x) + R_A x = 0$

$$M(x) = -R_A x = \frac{2}{3} qLx$$

- Till höger om B:



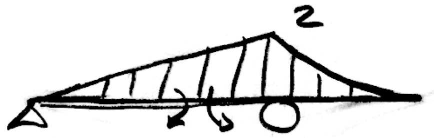
$\uparrow$: $-T(\xi) - q \cdot \xi = 0$

$$T(\xi) = -q \cdot \xi$$

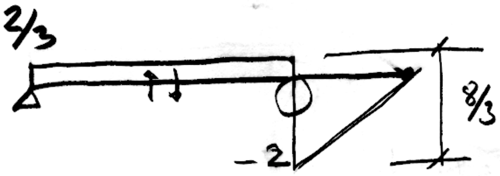
$\overset{\curvearrowright}{\xi}$: $-M(\xi) + q \cdot \xi \cdot \frac{\xi}{2} = 0$

$$M(\xi) = q\frac{\xi^2}{2}$$

Snittkraftsdiagram

M: 

$[\cdot qL^2]$

T 

$[\cdot qL]$

b) Största o minsta böjnormalspänning

Bestäm I:

$$c = h\cdot\frac{ht + ht/2}{ht + ht} = h\cdot\frac{\frac{3}{2}ht}{2ht} = \frac{3}{4}h$$

$$I_y = \frac{th^3}{12} + ht\left(\frac{3}{4}h - \frac{h}{2}\right)^2 + th\left(h - \frac{3}{4}h\right)^2$$

$$= \frac{th^3}{12} + ht\cdot\frac{1}{16}h^2 + \frac{1}{16}th^3$$

$$= th^3\left(\frac{1}{12} + \frac{2}{16}\right) = th^3\left(\frac{1}{12} + \frac{1}{8}\right)$$

$$= th^3\left(\frac{2+3}{24}\right) = \frac{5}{24}th^3$$

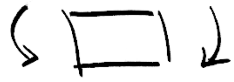
$$\sigma = \frac{M_y}{I_y}z \Rightarrow \sigma_{max} = \frac{M_{y,max}}{I_y}\cdot z_{max}$$

$$\sigma_{max} = \frac{2\cdot qL^2}{\frac{5}{24}th^3}\cdot(h-c) = \frac{48}{5}\cdot\frac{qL^2}{th^3}\left(\frac{h}{4}\right)$$

$$= \frac{12}{5}\frac{qL^2}{th^2} = \frac{12\cdot 2\cdot 10^3\cdot 4}{5\cdot 0{,}01\cdot 0{,}04} = 48 \text{ MPa}$$

$$\sigma_{min} = \frac{M_y}{I_y}\cdot z_{min} = \frac{M_y}{I_y}\cdot(-c) = -3\cdot 48 = -144 \text{ MPa}$$

3



a)

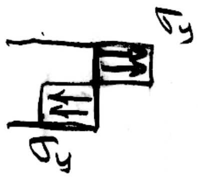
$$\sigma_{max} = \frac{M}{I} \cdot z_{max}$$

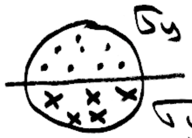
$$I = \frac{\pi d^4}{64} \quad , \quad z_{max} = \frac{d}{2}$$

$$\sigma_{max} = \frac{M \cdot 32}{\pi d^3} = \sigma_s \Rightarrow M_{max} = \underline{\underline{\frac{\pi d^3}{32} \sigma_s}}$$

$$\left[\frac{m^3 \cdot N}{m^2} = Nm \right]$$

b)

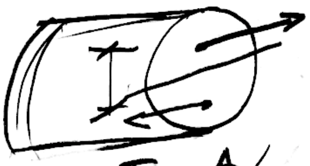




Dubbelsymmetriskt tvärsnitt:

Neutrallager i tyngdpunkt även efter plasticering.

Resultanten av spänningarna ger flytmomentet:



$$A = \frac{\pi d^2}{4}$$

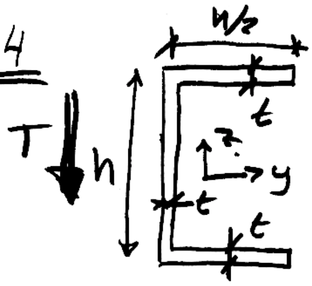
Resultanter i halvcirklarnas tyngdpunkt:

$$z_{TP} = \frac{2d}{3\pi}$$

$$M_s = \frac{\sigma_s \cdot \pi d^2}{8} \cdot 2 \cdot \frac{2d}{3\pi} = \underline{\underline{\frac{d^3}{6} \sigma_s}} \qquad \left[\frac{m^3 \cdot N}{m^2} = Nm \right]$$

$$\left(\beta = \frac{M_s}{M_e} - 1 = \frac{d^3 \, 32 \, \sigma_s}{6 \cdot \pi d^3 \, \sigma_s} - 1 = \frac{16}{3\pi} - 1 \approx 70\% \right)$$

4



$t \ll h$

sym ⇒

a) Största skjuvspänning vid $z = 0$

$$S_{A^*} = \frac{h}{2} \cdot t \cdot \frac{h}{2} + \frac{h}{2} \cdot t \cdot \frac{h}{4}$$
$$= \frac{h^2}{4} t + \frac{h^2}{8} t = \frac{3}{8} h^2 t$$

Bestäm I:

$$c = \frac{h}{2} \frac{h \cdot t + \frac{h}{2} t}{h \cdot t + 2 \frac{h}{2} \cdot t} = \frac{h}{2} \frac{3ht}{2 \cdot 2ht} = \underline{\underline{\frac{3}{8} h}}$$

$$I_y = \frac{th^3}{12} + \frac{1}{2} t \cdot \frac{h}{2} \cdot h^2 = th^3 \left(\frac{1}{12} + \frac{1}{4}\right)$$
$$= \frac{4}{12} th^3 = \underline{\underline{\frac{1}{3} th^3}}$$

$$\tau_{max} = \frac{T \cdot 3h^2 t \cdot 3}{8 \cdot th^3 \cdot t} = \frac{9Th^2 t}{8h^3 t^2} = \underline{\underline{\frac{9}{8} \frac{T}{ht}}} \left[\frac{N}{m^2}\right]$$