

HÅLLFASTHETSLÄRA I2

Konstruktionsuppgift 2

Balkböjning

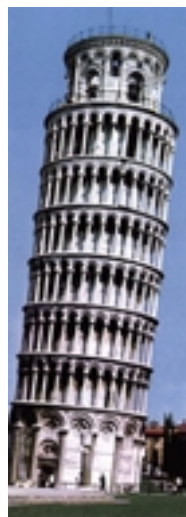
(av Peter Möller, fritt efter en idé av Anders Johansson)

Sista inlämningsdag 2001-11-14

ALLMÄNT

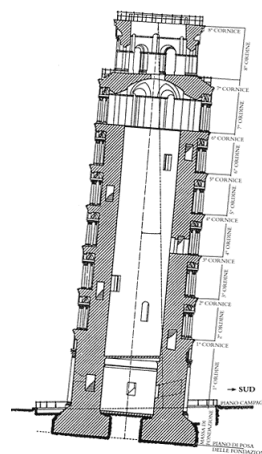
I en nyligen publicerad vetenskaplig artikel har man kommit fram till att lutande tornet i Pisa (Torre pendente di Pisa) kommer att falla då dess lutning uppgår till $6,9^\circ$ (se ZHENG ZHOU, MADARA OGOT & LILLIAN SCHWARTZ, **A finite element analysis of the effect of an increasing angle on the tower of Pisa**, *Finite Elements in Analysis and Design*, vol 37, no 11, sid 901–911 (oktober 2001)).

Efter att genomfört över 100 hållfasthetsberäkningar, var och en innefattande lösningen av ett ekvationssystem med ca 150000 ekvationer (och lika många obekanta), kom författarna fram till att vid denna vinkel uppkommer dragspänningar i väggarna – eftersom murverk bara kan ta upp obetydliga dragspänningar innan det spricker, faller då tornet.



UPPGIFTER

1. Beräkna vid vilken vinkel tornet välter (d v s dragspänningar uppkommer) enligt teknisk balkteori. Den lastbärande strukturen består av en murad cylinder med yttre diameter $D_1 \approx 15.5$ m och inre diameter $D_2 \approx 7.4$ m (gäller för tornets nedre delar). Övriga data som behövs för uppgiften hämtas från 'nätet' – se bifogade sidor.
2. År 1995 hängde man på ca 830 ton bly-plattor på ytterväggen nära tornets bas, se *figur 1*. Betrakta denna massa som en punktlast, $P = Mg$ och utred hur den påverkar den vinkel tornet kan tillåtas luta.
3. På sidan <http://www.newton.dep.anl.gov/askasci/phy99/phy99340.htm> får man, under rubriken "Ask A Scientist", veta att tornet i Pisa "will topple when the center of mass gets to the edge of its base". Vilket antagande måste göras för att detta ska vara riktigt?



figur 1 Lutande tornet i Pisa utan motvikter, med motvikter och sektionsritning

YTTERLIGARE DATA

The measurements of the Tower

The Tower is 58.36 metres high from the foundation and 55 from the ground. Its weight has been calculated at 14,453 tonnes. The centre of gravity is 22.6 metres above the foundation, which has an exterior dimension of 19.58 metres, with a central

aperture of 4.5 metres. The area of the annular foundation is thus 285m², and the average pressure on the ground is 497 kPa. The present inclination is about 55j - i.e. about 10%; the value corresponding to the eccentricity on the loads on the foundation is 2.3 metres.

(från http://torre.duomo.pisa.it/english_version/storia_misure.html)

Tower basics

- Official Name: TORRE PENDENTE DI PISA
- Function: Bell Tower (Campanile)
- Original Architect: Bonanno Pisano
- Years Built: 1173-1350
- Base Elevation above Sea Level: About 2 meters
- Latitude: 43.7167 (43° 43' 0" N)
- Longitude: 10.3833 (10° 22' 60" E)
- Altitude of Piazza dei Miracoli 9 feet, (2 meters) (DMS)
- First Construction Stop: 1178 (War with Firenze)
- Year in which lean became obvious: 1178 (Third Story)
- Height at which lean became obvious: 10.6 meters (35 ft.)
- Second Construction Stop: 1185 (War with Firenze)
- Later Construction Stop: 1284 (War with Genoa, Major Sea Battle Defeat)
- First Bells added: 1198 (Third Floor)
- Height: 55.863 meters (185 feet). 8 stories.
- Outer Diameter of Base: 15.484 meters
- Inner Diameter of Base: 7.368 meters
- Direction of Lean: 1173-1250 North, 1272-1997 South
- Architect who realized that the Leaning Tower could not be straightened : Tomasso di Andrea da Pontedera (1275)
- Level at which Tower Straightens to North: 5
- 7th Floor Completed: 1319
- Bell Tower Completed: 1350
- Number of Bells: 7, tuned to musical scale
- Largest Bell: Three and a half tons, cast in 1655.
- Oldest Bell's name: Pasquarreccia
- Address: Campo dei Miracoli - the "Field of Miracles", Pisa, Italia
- Weight: 14,700 metric tons
- Thickness of Walls at the Base: 8 feet
- Year cement injected into base, (blamed for lean acceleration): 1934
- Steps to Bell Tower: 294
- Number of visitors who climbed to top in 1989: 700,000
- Date Closed to Public: 7 January 1990
- Rate of Fall in 1990: 1.2 mm (1/20") every year ("Un millimetre per anno")
Source: The Guardian (London) August 19 1997
- Weight of Lead added on North side (picture above): 600 tons (1995)
- Amount of tilt recorded overnight in September 1995: 2.5 mm (0.07")
- Weight of Lead added after overnight tilt: 230 tons
- Amount of tilt correction since 1990: 25mm (1.0")

Lean Data Provided by Prof. Pierotti:

- 1.43 meters (1298) (Giovanni Pisano, reconstructed by P. Sanpaolesi)
- 3.79 meters (around 1550) (Vasari, six arms and half)
- 3.79 meters (1787 and 1812) (By Morrona, six arms and half)
- 3.84 meters (1817) (Crazy & Taylor, 12 feet and 7 thumbs)
- 4.04 meters (1911) (Pizzetti) (+ 20 cm Crazy & Taylor)
- The 1911 measure was taken from the outside of the seventh cornice with a theodolite (not from the inside with a plumb line) and therefore the resulting linear values are greater due to the thickness of the cornice, leaning relative to the body of the tower.
- Other Sources:
- Lean: 1.63 meters (1360)
- Lean: 1.43 meters (1372) The Guardian (London) August 19 1997
- Lean: 3.77 meters (1550)

- Lean: 4.75 meters (1817)
- Lean: 4.8 meters (1935)
- Lean: 5.2 meters (1997)

(från <http://www.endex.com/gf/buildings/lt/pisa/ltinfo.htm>)

Leaning Tower of Pisa

Question: Given the fact that the Leaning Tower of Pisa is increasing its lean $1/20$ every year how long will it take before it topples over?

Leaning Tower of Pies

Answer: It will topple when the center of mass gets to the edge of its base. If we knew the location of the center of mass (about half way up) and could determine how much further it needed to twist, we could make an estimate, but we would be assuming that the increase would be the same each year. It will be much more in the last few years.

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(från <http://www.newton.dep.anl.gov/askasci/phy99/phy99340.htm>)