Solutions to the written exam in *Fatigue design* for:

- IMP in Automotive Engineering
- IMP in Naval Architecture
- M4 0722 MMA115 (Utmattningsdimensionering)

THEORETICAL PART (14 p)

Question 1 (4 p)

Short crack

- $\Delta\sigma_{\text{nom}} = \Delta P/(2bt)$
- $\Delta K_{\text{I}} = \Delta\sigma_{\text{nom}} f(g)\sqrt{\pi 2a} = \dfrac{\Delta P}{2bt} f(g)\sqrt{\pi 2a} \approx 0.707\dfrac{\Delta P}{bt} f(g)\sqrt{\pi a}$

Long crack

- $\Delta\sigma_{\text{nom}} = \Delta P/(4bt)$
- $\Delta K_{\text{I}} = \Delta\sigma_{\text{nom}} f(g)\sqrt{\pi 4a} = \dfrac{\Delta P}{4bt} f(g)\sqrt{\pi 4a} = 0.5\dfrac{\Delta P}{bt} f(g)\sqrt{\pi a}$

Since the stress-intensity factor for the short crack is larger than the stress-intensity factor for the long crack, the former crack will grow the fastest.

Question 2 (3 p)

(1) Manufacturing process (due to e.g. residual stresses and their distribution).
(2) The geometry may differ from drawing to finished/manufactured product.
(3) The actual loads (or all of the loads) that the component is subjected to are not always known in detail in the (numerical) fatigue design analysis.

Question 3 (4 p)

Part (a):

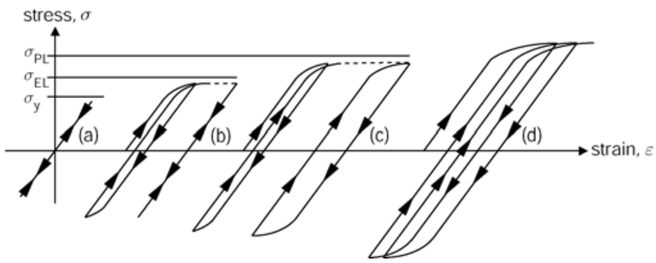
- *Stress-life (S-N) approach*: Homogeneous material, load levels that give elastic material response, or elastic shakedown material response. The number of cycles to initiation/failure is often high.
- *Stress-life approach*: Homogeneous material, load levels that give plastic shakedown or ratchetting material response. The number of cycles to initiation/failure is lower as compared with the stress-life approach.
- *Fracture mechanics approach*: A crack is present and crack growth in the Paris regime can be describe by the stress-intensity factor, which is a function of applied load level, geometry, and crack length. The approach can be used to calculate the number of cycles to complete fatigue failure. Also, since crack length gives a physical measure of fatigue damage, the approach can be used to find the "safe life" of cracked components.

Question 3 (4 p), continued …

Part (b):

(a) elastic material response
(b) elastic shakedown
(c) plastic shakedown
(d) ratchetting

(a) and (b): high-cycle fatigue (HCF)
(c) and (d): low-cycle fatigue (LCF)



Question 4 (3 p)

(a) Fracture will occur perpendicular to the maximum principal stress, i.e. the crack will propagate in the length direction of the hot dog.

(b) Initiation will occur on the plane of maximum shear stress (recall that according to definition, the hydrostatic stress is direction independent). This plane will be rotated 45 degrees from the maximum principal stress (circumferential) towards the plane of minimum principal stress (the radial direction). This can be seen also in Mohr's (3D) stress circle.

(c) In (*b*), the fatigue crack initiation is governed by the maximum shear stress (which governs the microscopic plastic deformations in stress-based approaches). In (*a*), presumed that the crack has grown some millimetres, its crack growth direction changes to be perpendicular to the maximum principal stress since this is the direction that results in the largest crack driving force (stress-intensity factor) divided by the resistance (crack friction and material strength).

PROBLEM PART (36 p)

Question 5 (12 p)

- Use the SWT approach (Dowling's book on p.394) to find σ_{ar} from $\sigma_{\max}$:

$$\sigma_{\text{ar}} = \sqrt{\sigma_{\max}\sigma_{\text{a}}} = \sqrt{(\sigma_{\max})^2/2} = \sigma_{\max}/\sqrt{2} \text{ when } \sigma_{\text{a}} = \sigma_{\max}/2.$$

- From the definition of $\sigma_{\max}$ in Figure A8 in Dowling's book on p.789:

$$\sigma_{\max} = \frac{P_{\max}}{(w-d)t} = \frac{24\cdot 10^3}{0.040\cdot 0.010\left(1-\frac{d}{w}\right)} \text{ Pa, i.e. } \sigma_{\text{ar}} = \frac{60\cdot 10^6}{\sqrt{2}\left(1-\frac{d}{w}\right)} \approx \frac{42.4\cdot 10^6}{\left(1-\frac{d}{w}\right)} \text{ Pa.}$$

- The endurance limit $\sigma_{\text{e}} = 300$ MPa has to be reduced by m, k_{f} and the safety factor $S = 2$. The allowed stress is then (using e.g. Eq. 10.33):

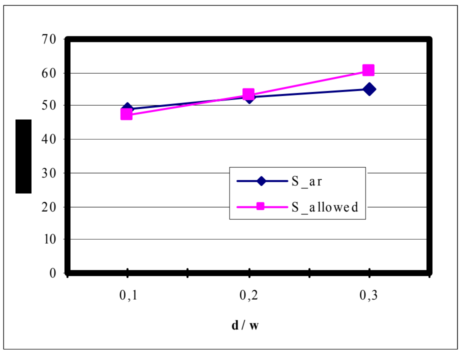
$$\sigma_{\text{allowed}} = \frac{\sigma_{\text{e}}\cdot m}{S\cdot k_{\text{f}}(d/w)} = \frac{300\cdot 0.85\cdot 10^6}{2\cdot k_{\text{f}}(d/w)} = \frac{127.5\cdot 10^6}{k_{\text{f}}(d/w)} \text{ Pa.}$$

- Peterson's formula (Eq. 10.6) gives that:

$$q = \left(1+\frac{\alpha}{(d/2)}\right)^{-1} \text{ where } \alpha = 0.025\left(\frac{2070}{640}\right)^{1.8} = 0.21 \text{ mm (Eq. 10.8).}$$

- Produce a table with k_{t} from Figure A8 (k_{f} from Eq. 10.8).

d/w	$r = d/2$ (mm)	q	$k_{\text{f}} = 1+q(k_{\text{t}}-1)$	σ_{allowed} (MPa)	σ_{ar} (MPa)
0.1	2	0.91	2.59	49.2	47.1
0.2	4	0.95	2.43	52.5	53.0
0.3	6	0.97	2.31	55.2	60.6



The curves cross at $d/w = 0.2$, i.e. $d = 8$ mm. However, the crossing is shallow meaning that a small change in conditions results in large changes in the hole diameter.

ANSWER: The largest allowed diameter of the hole is $d = 8$ mm.

Question 6 (12 p)

- Set up an equivalent stress (note that $R = 0$):

$$\Delta\sigma_{\text{eq}} = \frac{\Delta K_{\text{eq}}}{f(g)\sqrt{\pi \text{a}}} = \left[\sum_j \frac{(\Delta\sigma_j)^4}{4}\right]^{1/4} = \left[\frac{1}{4}\left((60)^4 + 2\cdot(50)^4 + (40)^4\right)\right]^{1/4} = 51.4 \text{ MPa}.$$

- For this case with a wide sheet, it can be assumed that $f(g) = 1.12$ (see Figure 8.12 in Dowling's book on p.301).
- Ensure that all of the loads are above the threshold, $K_{\text{th}} = 2 \text{ MPa}\sqrt{\text{m}}$:
  $\Delta K_{\min} = 40\cdot 1.12\sqrt{\pi\cdot 0.002}\ \text{MPa}\sqrt{\text{m}} = 3.55\ \text{MPa}\sqrt{\text{m}} > 2\ \text{MPa}\sqrt{\text{m}}$.
- Check the condition for maximum crack length at critical fracture:
  $K_{\text{Ic}} = \Delta K_{\max} \quad\rightarrow\quad 30 = 60\cdot 1.12\sqrt{\pi\cdot a_{\text{crit}}} \quad\rightarrow\quad a_{\text{crit}} = 0.0634 \text{ m}$.
- With the given crack growth law, $\frac{\text{d}a}{\text{d}N} = 10^{-11}\left(51.4\cdot 1.12\sqrt{\pi a}\right)^4$, one gets that

$$N = \int_{0.0020}^{0.0634} \frac{\text{d}a}{10^{-11}\left(51.4\cdot 1.12\sqrt{\pi}\right)^4\cdot a^2} = \frac{1}{10^{-11}\left(51.4\cdot 1.12\sqrt{\pi}\right)^4}\left[\frac{1}{0.0020} - \frac{1}{0.0634}\right] = 4.47\cdot 10^5.$$

- Hence, the number of allowed sequences is $\bar{N} = N/4 = 1.12\cdot 10^5 = 112000$.

ANSWER: The number of allowed sequences is 112000

Question 7 (12 p)

- It has been given that $b = -0.09$ in the Basquin equation, $\sigma_a = C\left(N_f\right)^b$.
- It is also given that $\bar{N}_f = 1100$ cycles when $\sigma_a = \sigma_e$.
- The fatigue life that is asked for should be calculated when:
  - $\sigma_{a,1} = \sigma_e$ and $0.7N_f$.
  - $\sigma_{a,2} = 1.1\sigma_e$ and $0.2N_f$.
  - $\sigma_{a,3} = 1.2\sigma_e$ and $0.1N_f$.
- Using the Basquin equation, $N_{f,i} = \left(\dfrac{\sigma_{a,i}}{C}\right)^{1/b}$.
- The Palmgren-Miner linear damage rule: $\sum \dfrac{n_i}{N_{f,i}} = 1$.
- Using the Palmgren-Miner rule gives the following relationship:

$$\frac{0.7N_f}{\left(\frac{\sigma_e}{C}\right)^{1/b}} + \frac{0.2N_f}{\left(\frac{1.1\cdot\sigma_e}{C}\right)^{1/b}} + \frac{0.1N_f}{\left(\frac{1.2\cdot\sigma_e}{C}\right)^{1/b}} = 1$$

$$\frac{N_f}{\left(\frac{\sigma_e}{C}\right)^{1/b}}\left(\frac{0.7}{1} + \frac{0.2}{(1.1)^{1/b}} + \frac{0.1}{(1.2)^{1/b}}\right) = 1$$

$$N_f = \frac{\left(\frac{\sigma_e}{C}\right)^{1/b}}{\left(\frac{0.7}{1} + \frac{0.2}{(1.1)^{1/b}} + \frac{0.1}{(1.2)^{1/b}}\right)}.$$

- The fraction $(\sigma_e/C)^{1/b}$ can be expressed in the given data using the Basquin equation: $\sigma_a = \sigma_e \;\rightarrow\; \sigma_e = C\left(\bar{N}_f\right)^b \;\rightarrow\; \dfrac{\sigma_e}{C} = \left(\bar{N}_f\right)^b$, i.e. $\left(\dfrac{\sigma_e}{C}\right)^{1/b} = 1100$.
- Thus,

$$N_f = \frac{1100}{\left(\frac{0.7}{1} + \frac{0.2}{(1.1)^{1/-0.09}} + \frac{0.1}{(1.2)^{1/-0.09}}\right)} = 540 \text{ cycles.}$$

- Solving for fatigue life gives $N_f \approx 540$ cycles.

ANSWER: The number cycles to failure is approximately 540 cycles.