Written examination in *Fatigue design* for:

- IMP in Automotive Engineering
- IMP in Naval Architecture
- M4 0722 MMA115 (Utmattningsdimensionering)

- **Date:** Thursday 2004-08-26
- **Time:** 08.45–12.45
- **Location:** M
- **Teacher:** Jonas Ringsberg, phone: (772) 1504 or 0709 – 458666
- **Solutions** will be available 2004-08-26 at 13.00 at the Department of Applied Mechanics
- **Corrected examinations** will be available 2004-08-30 at the Department of Applied Mechanics

The written examination consists of two parts: one *theoretical part* of totally 14 points (14 p), and one *problem solving part* consisting of three problems of 12 points (12 p) each. A fully correct solution of the complete exam gives a maximum of 50 points.

Theoretical part

This part must be solved first. **No aids are allowed**, i.e. this part is to be solved without aids like textbooks or hand calculator. The solutions to this part should be handed in separately, before the student continues with the problem part.

Problem part

This part may be solved using the textbooks and text material distributed during the course, see below. These aids may be collected when the solutions to the theoretical part have been handed in. Note, however, that solved problems to exercises from the course are NOT allowed as aids. A hand calculator may be used. When in doubt about the hand calculator being a full computer, the teacher should be contacted to permit the hand calculator. **NOTE: To obtain maximum points for each problem, the solutions must be clearly motivated and all the equations used from the literature should have a clear reference.**

Allowed literature during the problem part

- Dowling, N.E. *Mechanical behaviour of materials, second edition*. Prentice Hall, Upper Saddle River, NJ, U.S.A., 1999.
- Fatigue design (MMA 115): Course material on multiaxial fatigue
  - Extract from Socie, D.F. and Marquis, G.B. *Multiaxial fatigue*. Society of Automotive Engineers Inc., Warrendale, PA, U.S.A., 2000.
  - *Multiaxial fatigue*. A compendium written by Anders Ekberg, Department of Solid Mechanics, Chalmers.
- Mathematical tables.
- Handbooks in engineering like *KTH's formelsamling i hållfasthetslära*, or Roark, R.J. and Young, W.C. *Formulas for stress and strain*.

The marks are given according to the following scheme (the sum of the points from the theoretical and the problem parts):

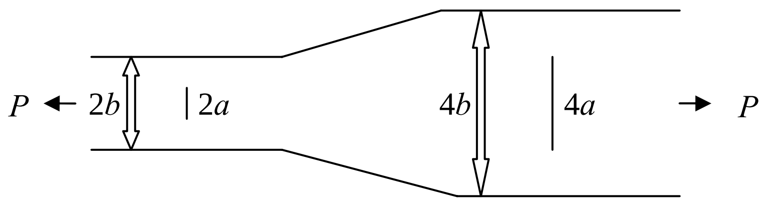
- Not passed 0–19 points
- 3 = passed 20–29 points
- 4 = very good 30–39 points
- 5 = excellent 40–50 points

THEORETICAL PART (14 p)

Question 1 (3 p)

A strip with uniform thickness t but with varying width according to the figure is loaded by a zero to tension load with maximum value P. Show by a simple analysis which of the cracks that will grow the fastest, the one with length $2a$ or the one with length $4a$.

It may be assumed that $b >> a$, LEFM is valid, the strip is so long that the cracks are positioned in uniform stress fields, and the threshold value for crack propagations is exceeded at both positions.



Question 2 (4 p)

In fatigue design there are three approaches to design against fatigue: (i) the stress-life approach, (ii) the strain-life approach, and (iii) the fracture mechanics approach. Explain when each of these approaches can be employed and what distinguishes them. Discuss in terms of load level, plastic material deformation, elastic material deformation, homogeneous or cracked material, etc.

Question 3 (3 p)

Describe the Palmgren-Miner summation rule for accumulation of fatigue damage and give at least one reason for its physical inconsistency.

Question 4 (4 p)

Describe how the da/dN vs. ΔK curve is constructed from an experiment that gives a diagram showing crack length as a function of cycles.

PROBLEM PART (36 p)

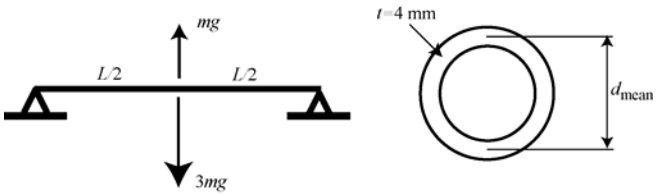
NOTE: To obtain maximum points for each problem, the solutions must be clearly motivated and all the equations used from the literature should have a clear reference (author, page and equation number).

Question 5 (12 p)

In the gymnastics part of the Olympic Games in Athens there is a men's exercise called the Horizontal bar.

Assuming that the maximum load causing risk for fatigue initiation after at least 10^6 cycles, i.e. when the gymnast is swinging around the bar, is a vertical load from *mg* upwards to 3*mg* downwards, calculate the outer diameter *d* of the bar necessary to give a safety factor of 2 against such an occasion. Use the Juvinall approach, the Morrow correction for mean value influence, and the solid mechanics formulae of Appendix A in Dowling's book. The product of the correction factors for surface finish, dimension etc, may be assumed to be 0.85.

Assume a circular tube of wall thickness 4 mm and length 2.4 meters, freely supported at the ends (no bending moments at the ends) and loaded in the middle, and made of the material AISI 4340 (Table 9.1 in Dowling's book). The mass *m* of the gymnast is assumed to be 60 kg and g = 9.81 m/s^2. In the calculations, the tube may be assumed to be thin-walled.



Question 6 (12 p)

For aluminium alloy 2024-T351 with strain range 0.016, do the following.

- Estimate the life from the Morrow equation for
  (a) σ_m = +70 MPa, and
  (b) σ_m = -70 MPa.

- Estimate the life using the Smith-Watson-Topper (SWT) parameter for
  (c) σ_m = +70 MPa, and
  (d) σ_m = -70 MPa.

- Finally in (e), comment on the trends in values. Also, comment on when the SWT parameter cannot be employed.

Table: Fatigue data for the aluminium alloy 2024-T351.

$\sigma_f' = 927$ MPa $\varepsilon_f' = 0.409$	$b = -0.113$ $c = -0.713$	$E = 73.1$ GPa $H' = 662$ MPa	$n' = 0.070$

Question 7 (12 p)

A wide sheet in an aircraft structure loaded by a uniform stress $\sigma(t)$ may develop edge cracks according to the figure below. The stress varies according to the stress history below and then this sequence is repeated.

How many sequences can be allowed between inspections?

Cracks larger than $a = 2$ mm can be detected and the maximum stress-intensity factor allowed is $K_{Ic} = 30\ \text{MPa}\sqrt{\text{m}}$. The yield stress is $\sigma_y = 600$ MPa, and the threshold value for crack growth is $K_{th} = 2\ \text{MPa}\sqrt{\text{m}}$ at $R = 0$. For the material, the relationship $\text{d}a/\text{d}N = 10^{-11}(\Delta K)^4$ [m/cycle] may be used with ΔK in units of $\text{MPa}\sqrt{\text{m}}$.