Solutions to the written exam in *Fatigue design* for:

- IMP in Automotive Engineering
- IMP in Naval Architecture
- M4 0722 MMA115 (Utmattningsdimensionering)

THEORETICAL PART (14 p)

Question 1 (3 p)

Short crack

- $\Delta\sigma_{\text{nom}} = \Delta P/(2bt)$
- $\Delta K_{\text{I}} = \Delta\sigma_{\text{nom}} f(g)\sqrt{\pi 2a} = \dfrac{\Delta P}{2bt} f(g)\sqrt{\pi 2a} \approx 0.707\dfrac{\Delta P}{bt} f(g)\sqrt{\pi a}$

Long crack

- $\Delta\sigma_{\text{nom}} = \Delta P/(4bt)$
- $\Delta K_{\text{I}} = \Delta\sigma_{\text{nom}} f(g)\sqrt{\pi 4a} = \dfrac{\Delta P}{4bt} f(g)\sqrt{\pi 4a} = 0.5\dfrac{\Delta P}{bt} f(g)\sqrt{\pi a}$

Since the stress-intensity factor for the short crack is larger than the stress-intensity factor for the long crack, the former crack will grow the fastest.

Question 2 (4 p)

- *Stress-life (S-N) approach*: Homogeneous material, load levels that give elastic material response, or elastic shakedown material response. The number of cycles to initiation/failure is often high.
- *Stress-life approach*: Homogeneous material, load levels that give plastic shakedown or ratchetting material response. The number of cycles to initiation/failure is lower as compared with the stress-life approach.
- *Fracture mechanics approach*: A crack is present and crack growth in the Paris regime can be describe by the stress-intensity factor, which is a function of applied load level, geometry, and crack length. The approach can be used to calculate the number of cycles to complete fatigue failure. Also, since crack length gives a physical measure of fatigue damage, the approach can be used to find the "safe life" of cracked components.

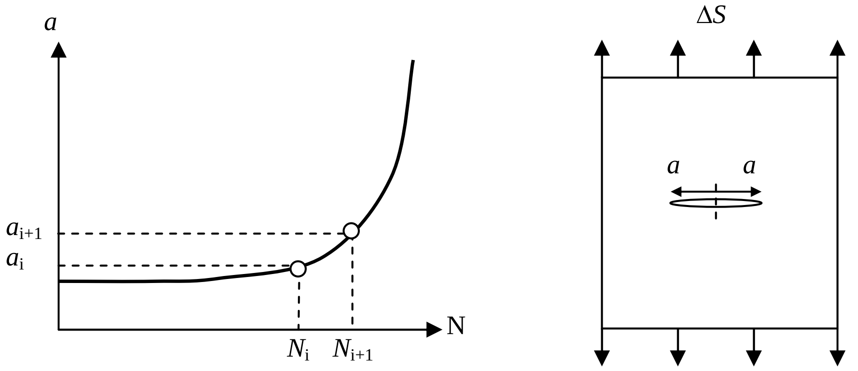
Question 3 (3 p)

The linear damage rule defines a cycle ratio n/N where n is the number of cycles at a stress level S and N is the fatigue life in cycles at stress level S. The damage fraction, D, is defined as the fraction in life used up by an event or a series of events. Failure in any of the cumulative damage theories is assumed to occur when the summation of damage fractions equals 1, or $\sum D_i \geq 1$.

The linear damage rule states that the damage fraction, D, at stress level S_i is equal to the cycle ratio n_i / N_i. For example, the damage fraction, D, due to one cycle of loading is $1/N$. In other words, the application of one cycle of loading consumes $1/N$ of the fatigue life. The failure criterion for variable amplitude loading can now be stated as $\sum n_i / N_i \geq 1$. The life to failure can be estimated by summing the percentage of life used up at each stress level.

The linear damage rule has two main shortcomings when it comes to describing observed material behaviour. First, it does not consider sequence effects. The theory predicts that the damage caused by a stress cycle is independent of where it occurs in the load history. An example of this discrepancy is high-low and low-high fatigue tests. Second, the linear damage rule is amplitude independent. It predicts that the rate of damage accumulation is independent of stress level. The last trend does not correspond to observed behaviour. At high strain amplitudes cracks will initiate in a few cycles, whereas at low strain amplitudes almost all of the life is spent initiating a crack.

Question 4 (4 p)



The crack length is observed as a function of the number of cycles at certain intervals, see figure. The relationship between the load variation (ΔS), the crack length (a) and the stress intensity factor (ΔK) for the actual geometry is calculated as $\Delta K = f(a, \text{geometry})\Delta S\sqrt{\pi a}$. From the diagram, for the corresponding crack length, $\mathrm{d}a/\mathrm{d}N \approx (a_{\mathrm{i+1}} - a_{\mathrm{i}})/(N_{\mathrm{i+1}} - N_{\mathrm{i}})$ is evaluated (in various fashions). Then, for a number of corresponding points on the curve $\mathrm{d}a/\mathrm{d}N$, $\log(\mathrm{d}a/\mathrm{d}N)$ can be plotted as a function of $\log(\Delta K)$.

PROBLEM PART (36 p)

Question 5 (12 p)

- Material 4340 according to Table 9.1: $\sigma_u = 1172$ MPa; $\sigma'_f = 1758$ MPa .
- The Juvinall scheme for $N \geq 10^6$ cycles:
  $S_{er} = \dfrac{\sigma_{er} \cdot m}{K_f \cdot S_{safety}} = \dfrac{0.5\sigma_u \cdot 0.85}{1 \cdot 2} = 249$ MPa
- Morrow correction, with $\sigma_a = 2\sigma_m$ (since $P = mg \pm 2mg$):
  $S_{er} = 249 = \dfrac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma'_f}} = \dfrac{2\sigma_m}{1 - \dfrac{\sigma_m}{1750}}$ (in MPa) gives that $\sigma_m = 116$ MPa .
- $\sigma_m = \dfrac{P_m L/2}{W_b} = \dfrac{mgL/2}{\dfrac{\pi}{4} d_{mean}^2 t} \rightarrow d_{mean} = \sqrt{\dfrac{2mgL}{\pi \sigma_m t}} = 44$ mm .
- Therefore, $d = d_{mean} + t = 48$ mm .

ANSWER:
$d = d_{mean} + t = 48$ mm

Question 6 (12 p)

- The material data for the aluminium was given in a table as:

$\sigma'_\text{f} = 927$ MPa $\varepsilon'_\text{f} = 0.409$	$b = -0.113$ $c = -0.713$	$E = 73.1$ GPa $H' = 662$ MPa	$n' = 0.070$

- The component was subjected to the strain range $\Delta\varepsilon = 0.016$.

Part (a).

- Use $\varepsilon_\text{a} = 0.008$, $\sigma_\text{m} = 70$ MPa , and the Morrow equation (Dowling eq. 14.18) to calculate $N_\text{f,a}$.
- Start with $\varepsilon_\text{a} = \frac{\sigma'_\text{f}}{E}\left(2N^*\right)^b + \varepsilon'_\text{f}\left(2N^*\right)^c$. Numerical solution gives $N^* = 679$ cycles .
- The Morrow equation gives $N_\text{f,a} = N^*\left(1-\frac{\sigma_\text{m}}{\sigma'_\text{f}}\right)^{-1/b} = 339$ cycles .

Part (b).

- Use $\varepsilon_\text{a} = 0.008$, $\sigma_\text{m} = -70$ MPa , and the Morrow equation (Dowling eq. 14.18) to calculate $N_\text{f,b}$.
- Start with $\varepsilon_\text{a} = \frac{\sigma'_\text{f}}{E}\left(2N^*\right)^b + \varepsilon'_\text{f}\left(2N^*\right)^c$. Numerical solution gives $N^* = 679$ cycles .
- The Morrow equation gives $N_\text{f,b} = N^*\left(1-\frac{\sigma_\text{m}}{\sigma'_\text{f}}\right)^{-1/b} = 1293$ cycles .

Part (c).

- Use $\varepsilon_\text{a} = 0.008$, $\sigma_\text{m} = 70$ MPa , and the SWT relationship (Dowling eq. 14.26) to calculate $N_\text{f,c}$.
- SWT relationship: $\sigma_\text{max}\varepsilon_\text{a} = \frac{\left(\sigma'_\text{f}\right)^2}{E}\left(2N_\text{f}\right)^{2b} + \varepsilon'_\text{f}\sigma'_\text{f}\left(2N_\text{f}\right)^{b+c}$.
- Here, $\sigma_\text{max} = \sigma_\text{a} + \sigma_\text{m}$, i.e. we must calculate σ_a .
- σ_a can be calculated using $\varepsilon_\text{a} = \frac{\sigma_\text{a}}{E} + \left(\frac{\sigma_\text{a}}{H'}\right)^{1/n'}$ (Dowling eq. 14.1).
- Substitute ε_a and constants and solve numerically for $\sigma_\text{a} = 430$ MPa :
  $\sigma_\text{a} = 430$ MPa $\rightarrow$ $\sigma_\text{max} = 500$ MPa, $\sigma_\text{max}\varepsilon_\text{a} = 4.00$ MPa .
- Substitute $\sigma_\text{max}\varepsilon_\text{a}$ and constants in the SWT relationship and solve numerically for $N_\text{f,c} = 426$ cycles .

Part (d).

- Use $\varepsilon_{\mathrm{a}} = 0.008$, $\sigma_{\mathrm{m}} = -70$ MPa, and the SWT relationship (Dowling eq. 14.26) to calculate $N_{\mathrm{f,c}}$.
- Same procedure as in Part (c) above but with the new σ_{m} value:
  $\sigma_{\mathrm{a}} = 430$ MPa $\rightarrow$ $\sigma_{\max} = 360$ MPa, $\sigma_{\max}\varepsilon_{\mathrm{a}} = 2.88$ MPa.
- Substitute $\sigma_{\max}\varepsilon_{\mathrm{a}}$ and constants in the SWT relationship and solve numerically for $N_{\mathrm{f,c}} = 947$ cycles.

Part (e).

The lives differ between Morrow and SWT, but both show a considerable increase in changing $\sigma_{\mathrm{m}} = 70$ MPa to $\sigma_{\mathrm{m}} = -70$ MPa. The SWT parameter cannot be used when $\sigma_{\max} < 0$.

ANSWER:
(a). $N_{\mathrm{f,a}}$ = 339 cycles.
(b). $N_{\mathrm{f,b}}$ = 1293 cycles.
(c). $N_{\mathrm{f,c}}$ = 426 cycles.
(d). $N_{\mathrm{f,d}}$ = 947 cycles.
(e). See the text above.

Question 7 (12 p)

- Set up an equivalent stress (note that R = 0):

$$\Delta\sigma_{\mathrm{eq}} = \frac{\Delta K_{\mathrm{eq}}}{f(g)\sqrt{\pi \mathrm{a}}} = \left[\sum_j \frac{(\Delta\sigma_j)^4}{4}\right]^{1/4} = \left[\frac{1}{4}\left((60)^4 + 2\cdot(50)^4 + (40)^4\right)\right]^{1/4} = 51.4 \text{ MPa} .$$

- For this case with a wide sheet, it can be assumed that $f(g) = 1.12$ (see Figure 8.12 in Dowling's book on p.301).
- Ensure that all of the loads are above the threshold, $K_{\mathrm{th}} = 2 \text{ MPa}\sqrt{\mathrm{m}}$:
  $\Delta K_{\min} = 40\cdot 1.12\sqrt{\pi\cdot 0.002} \text{ MPa}\sqrt{\mathrm{m}} = 3.55 \text{ MPa}\sqrt{\mathrm{m}} > 2 \text{ MPa}\sqrt{\mathrm{m}}$.
- Check the condition for maximum crack length at critical fracture:
  $K_{\mathrm{Ic}} = \Delta K_{\max} \quad\rightarrow\quad 30 = 60\cdot 1.12\sqrt{\pi\cdot a_{\mathrm{crit}}} \quad\rightarrow\quad a_{\mathrm{crit}} = 0.0634 \text{ m}$.
- With the given crack growth law, $\frac{\mathrm{d}a}{\mathrm{d}N} = 10^{-11}\left(51.4\cdot 1.12\sqrt{\pi a}\right)^4$, one gets that

$$N = \int_{0.0020}^{0.0634} \frac{\mathrm{d}a}{10^{-11}\left(51.4\cdot 1.12\sqrt{\pi}\right)^4\cdot a^2} = \frac{1}{10^{-11}\left(51.4\cdot 1.12\sqrt{\pi}\right)^4}\left[\frac{1}{0.0020} - \frac{1}{0.0634}\right] = 4.47\cdot 10^5.$$

- Hence, the number of allowed sequences is $\bar{N} = N/4 = 1.12\cdot 10^5 = 112000$.

ANSWER: The number of allowed sequences is 112000