Solutions to the written examination in *Fatigue Design* for:
* IMP in Automotive Engineering
* IMP in Naval Architecture
* M4 0722 MMA115 (Utmattningsdimensionering)

THEORETICAL PART (14 P)

Question 1 (4 p)

a) See Dowling p 422-423. The stress in a notched member decreases rapidly with increasing distance from the notch. The material is not sensitive to peak stress, but rather to the average stress that acts over a region of small but finite size, called the process zone δ. The stress that controls the initiation of fatigue damage is the average out to x = δ. This average stress is then expected to be the same as the smooth specimen fatigue limit σe. Hence

k_f = (average σ_y out to $x = \delta$) / $S_a = \sigma_e$ / S_a

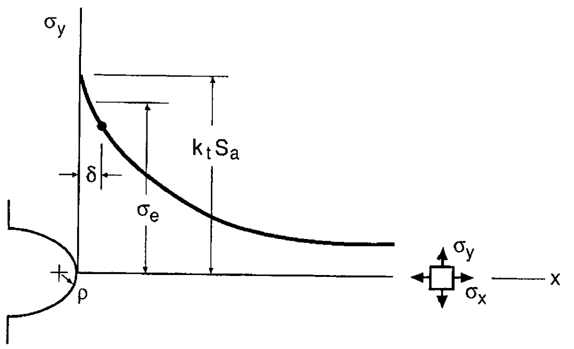


Figure 10.3 Interpretation of the fatigue limit as the average stress over a finite distance δ ahead of the notch.

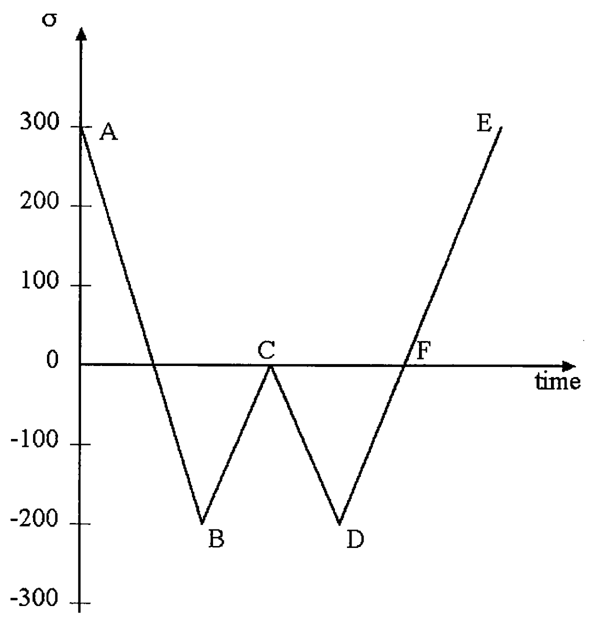
b) See Dowling p 423-424. The crack may start quickly in a sharply notched member during cyclic loading so that the fatigue behaviour is dominated by crack growth. Consider cyclic loading of a notched member under the nominal stress *S*. Then let a smooth member be cycled under a stress σ_s that is the same as that at the notch in the notched member $\sigma_s = k_t \cdot S$. Based on the stress concentration one would expect the same fatigue life. However in the notched member the crack is growing into a region of rapidly decreasing stress as seen in the Figure above. Hence more cycles are required to grow the crack than in the smooth specimen which gives $k_f < k_t$,

Question 2 (3 p)

See Dowling p 492-493. When K_{max} approaches K_{IC} crack growth is faster than given by Paris' law, i e the final fracture occurs for fewer cycles than predicted. Hence the solution is non-conservative.

Question 3 (3 p)

Rainflow count is described in Dowling p 404-406. For the sequence given one has



Rainflow counting now gives the stress cycles C-D-F and A-B-E with $\sigma_a = 100,\ 250$ and $\sigma_m = -100,\ 50$, respectively.

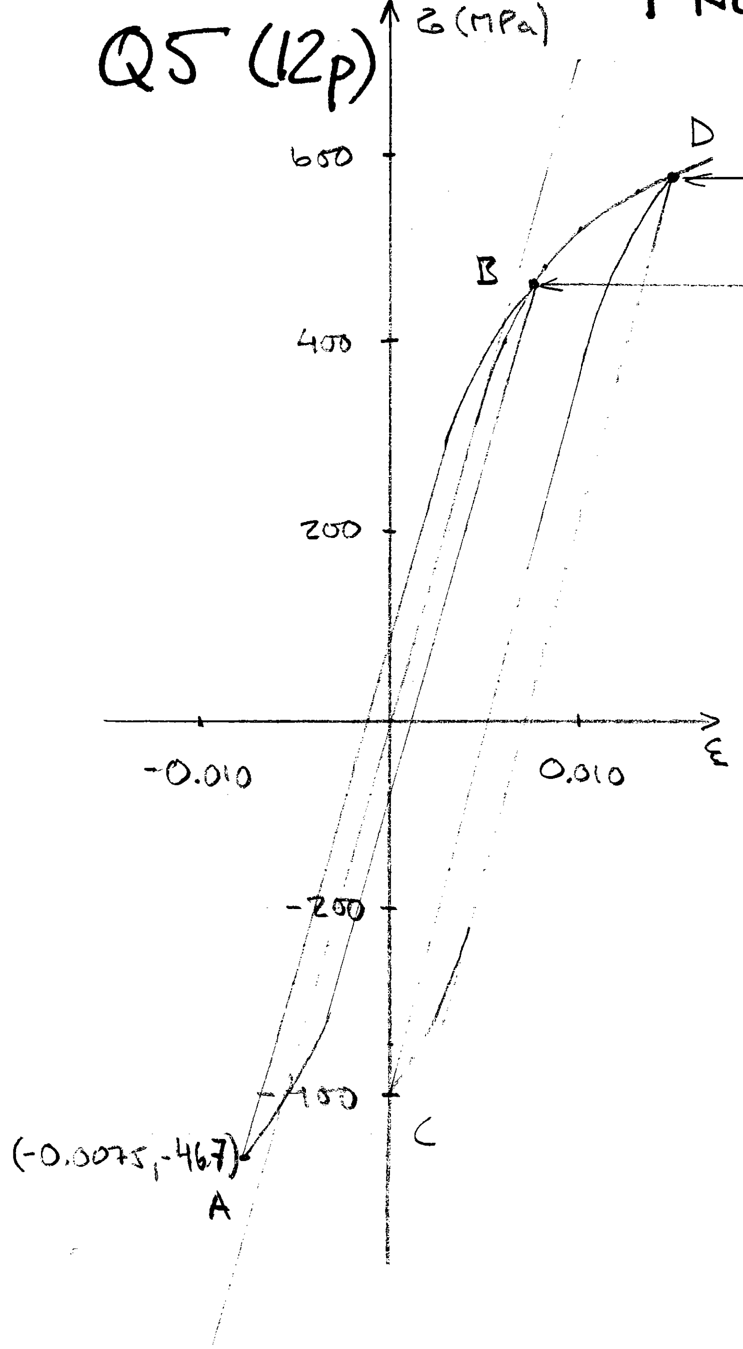
Question 4 (4 p)

Fatigue of welded joints is described in Dowling p 458 – 459. Welds are sensitive to fatigue due to

- There will be a strong geometric discontinuity at the weld and hence a high stress concentration factor and fatigue notch factor. Investigations of, in particular, filled welded joints show that macroscopic cracks, with length of 0.1 – 0.5 mm are found at the weld toe.
- In the welding process, the material is rapidly heated to a high temperature followed by rapid cooling leading to local yielding first in compression then during cooling in tension. The material will therefore as a result contain (self-equilibring) residual stresses with high tensile magnitudes at the weld balanced by compressive stresses away from the weld region. The material at the weld toe (in a filled weld) will therefore be subject to the sum of a cyclic external stress (with stress range $\Delta\sigma$) and a (high) tensile mean stress. Conservatively, one may assume that the material at the weld toe is subject to a stress that varies between σ_{yield} and $\sigma_{\text{yield}} - \Delta\sigma$ irrespective of the stress ratio R of the external stress. This high mean stress will reduce the fatigue life.

PROBLEM PART (36 P)

Q5 (12p)



USE OF THE CYCLIC STRESS STRAIN DATA GIVES

$$\varepsilon_a = \frac{\sigma_a}{71000} + \left(\frac{\sigma_a}{977}\right)^{1/0.106}$$

FOR $\varepsilon_a = 0.0075$ σ_a IS OBTAINED AS $\sigma_a = 467$ MPa, AND THE INTENDED LOOP WOULD BE FROM A TO B.

FOR $\varepsilon_a = 0.0150$ ONE GETS $\sigma_a = 576$. SINCE THIS LOOP WILL HAVE THE SAME SIZE AND SHAPE (SAME $\Delta\varepsilon$), THE STRESS RANGE WILL BE THE SAME, SO THE STRESS AT C WILL BE $576 - 2 \cdot 467 = -358$. THEN THE MEAN STRESS WILL BE $(576 - 358)/2 = 109$ MPa

MORROW'S EQUATION GIVES

$$N_f = N^* \left(1 - \sigma_m/\sigma_f'\right)^{-1/b} = N^*\left(1 - \frac{109}{1466}\right)^{\frac{1}{0.143}} = N^* \cdot 0.58$$

SO N_f WILL BE ONLY AROUND 60% OF THE CORRECT LIFE ESTIMATE.

Q6 (12p) WITH THE CRACK PRESENT THE REMAINING LIFE WILL BE, AT MOST:

$$N = \frac{a_f^{1-\frac{3.2}{2}} - a_i^{1-\frac{3.2}{2}}}{5\cdot10^{-13}(1.12\cdot140\sqrt{\pi})^{3.2}(1-\frac{3.2}{2})}$$

WITH $a_i = 0.010$ M AND a_f FROM

$$K_{Ic} = 1.12\cdot140\sqrt{\pi a_f} \Rightarrow a_f = \frac{60^2}{\pi\, 1.12^2\, 140^2} = 0.0466$$

IE $$N \leq \frac{0.010^{-0.6} - 0.047^{-0.6}}{0.6\cdot5\cdot10^{-13}(1.12\cdot140\sqrt{\pi})^{3.2}} = \frac{15.85 - 6.29}{3\cdot10^{-13}\cdot66.2\cdot10^{6}} =$$

$= 4.8\cdot10^5$ CYCLES. (LEFM CONDITION FULFILLED!)

WITH THE HOLE PRESENT THE LIFE IS ESTIMATED BY

$$\sigma_{ar} = \sigma_f'(2N)^{b}, \text{ IF } \sigma_{ar} > \sigma_{er}$$

WHERE $\sigma_{ar} = S_{ar}K_f$ AND (SWT) $S_{ar} = \sqrt{S_a(S_a+S_m)}$

SINCE $S_a = S_m = S_{MAX}/2$ $S_{ar} = \sqrt{\frac{S_{MAX}}{2} S_{MAX}} = S_{MAX}/\sqrt{2}$

IE $\sigma_{ar} = 4.2\frac{1}{\sqrt{2}}S_{max} = 416$ MPa

SINCE $\sigma_{YIELD} < \sigma_u = 1200$ AND IT CAN GENERALLY BE ASSUMED THAT $\sigma_{er} \approx 0.5\sigma_u > 600$ IN THIS CASE IT IS PROBABLE THAT THERE WILL BE NO INITIATION OF A NEW FATIGUE CRACK AT THE HOLE.

(IF NOT, A LIFE ESTIMATE WILL BE

$$N = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{416}{1800}\right)^{-10} = 1.15\cdot10^6 \quad)$$

Question 7 (12 p)

See the Course material on Multi-axial fatigue, p 12 -13 and 16-17. The Dang Van equivalent stress σ_{EQDV} and the criterion for initiation of fatigue damage is given as

$$\sigma_{EQDV} = \tau_{Trescaa}(t) + c_{DV}\sigma_h(t) > \sigma_{eDV}$$

where σ_{eDV} is the fatigue limit

Note that as discussed in the course material, a superposed hydrostatic stress will not influence the Tresca shear stress.

$$\frac{250--250}{2} = \sigma_{eDV} \Rightarrow \sigma_{eDV} = 250$$

$$\frac{400}{2} + c_{DV}\frac{400}{3} = \sigma_{eDV} \Rightarrow c_{DV} = \frac{50\cdot 3}{400} = 0.375$$

$$\sigma_{1,2} = \frac{300}{2} \pm \sqrt{\left(\frac{300}{2}\right)^2 + 100^2} = 330,-30$$

$$\sigma_{EQDV} = \frac{330--30}{2} + 0.375\cdot\frac{300}{3} = 217.5$$

$$SF_{DV} = \frac{250}{218} = 1.15$$

with residual stress $\sigma_x = \sigma_\varphi = -150$ MPa

$$\sigma_{EQDV} = \frac{330--30}{2} + 0.375\cdot\frac{(150-150+0)}{3} = 180\text{MPa}$$

$$SF_C = \frac{250}{180} = 1.4$$