Solutions to the written examination in *Fatigue Design* for:
* IMP in Automotive Engineering
* IMP in Naval Architecture
* M4 0722 MMA115 (Utmattningsdimensionering)

THEORETICAL PART (14 P)

Question 1 (4 p)
a) A larger notch root radius, *r*, means that increased stresses prevail deeper into the material, facilitating growth of an initiated crack. Hence, larger *r*-values are expected to result in higher K_f-values (closer to K_t). This is also seen from the structure of the expression of the notch sensitivity q.

b) Log $5 \cdot 10^4 = 4.70$, $2\sigma = 0.400$

Hence Log $N_{97.5} = 4.70 - 0.40 = 4.30 \Rightarrow N_{97.5} \approx 2 \cdot 10^4$

Question 2 (3 p)
There are crack closure effects at repeated loading in the growing crack tip (plasticity, environment etc) meaning that only part of ΔK_I is effective in the crack growth process. When R increases $K_{I,min}$ increases and the closure effect becomes smaller. Then more of ΔK_I is effective and da/dN increases.

Question 3 (3 p)
Rainflow count is described in Dowling p 404-406. Rainflow counting now gives four strain cycles for the present sequence with

Cycle	Mid strain ε_{mid}	Amplitude strain ε_{amp}
A-D-A	0	4.5
B-C-B	0.5	2
E-F-E	2.5	1
G-H-G	-2	0.5

Question 4 (4 p)
a) See Dowling p 671. The critical plane concept is an approach used in numerical calculations to find the material plane where the largest fatigue damage, according to a fatigue criterion, occurs during cyclic loading. In principle, the fatigue damage is calculated on a large number of material planes, each of them having different orientations. The plane that experiences the largest fatigue damage is identified as the "critical plane". In a multi-axial fatigue situation, the principal stress and strain directions vary in time (in-phase, out-of-phase, proportional or non-proportional loading). The critical plane must be used to find the material plane that experiences the largest fatigue damage. This is particularly important for elasto-plastic material responses.

b) See Dowling, p 402: The linear damage rule defines a cycle ratio n_i/N_i where n_i is the number of cycles at stress level S_i and N_i is the fatigue life at S_i. The damage fraction D_i is defined as the fraction in life used up at this stress level, $D_i = n_i/N_i$. Failure is assumed to occur when $\Sigma_i D_i = 1$
The linear (Palmgren-Miner) damage rule can not account for sequence effects.

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Q5 (12p)

For case a) the sequence is just one cycle with $\sigma_m = 100$ and $\sigma_a = 50$ MPa.

SWT, Dowling p 394 →

$$\sigma_{ar} = \sqrt{\sigma_{max} \cdot \sigma_a} = \sqrt{150 \cdot 50} = 87 \text{ MPa} < \sigma_{er}$$

→ very long life in most cases.

For case b) we have two cycles with
$\sigma_m = 75$ and $\sigma_a = 125$ MPa
$\sigma_m = 75$ and $\sigma_a = 25$ MPa respectively
the second one is insignificant, hence

$$\text{SWT} \Rightarrow \sigma_{ar} = \sqrt{200 \cdot 125} = 158 \text{ MPa}$$

(Second cycle $\sigma_{ar} = \sqrt{25 \cdot 100} = 50 \text{ MPa} < \sigma_{er}$)

The expected life can be estimated from

$$\sigma_{ar} = 430 \cdot N^{-0.1}$$

$$\log \sigma_{ar} = \log 430 - 0.1 \log N \rightarrow$$

$$\log N = \frac{\log 430 - \log 158}{0.1} = \frac{2.633 - 2.199}{0.1} = 4.34$$

$N = 22\,000$ sequences

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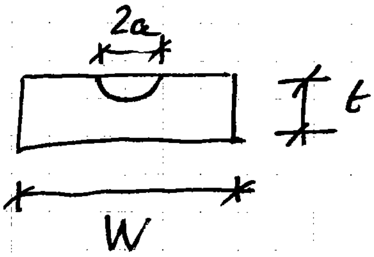
Q6 (12p)

Use Dowling Fig 8.17 b for geometry.
assume crack radius a small



$$K_{IP} = \frac{P}{Wt} F\sqrt{\pi a} =$$

$$= \frac{45 \cdot 10^3 \, 0.728}{45 \cdot 10} \sqrt{\pi a} = 72.8 \sqrt{\pi a}$$

$$K_{IM} = \frac{6M}{Wt^2} F\sqrt{\pi a} = \frac{6 \cdot 100 \cdot 10^3 \cdot 0.728}{10^2 \cdot 45} \sqrt{\pi a} = 97 \sqrt{\pi a}$$

Final crack length a_f determined from

$K_{max} = K_{IP} + K_{IM} \leq K_{Ic}/3$ hence

$$(72.8 + 97)\sqrt{\pi a_f} \leq \frac{60}{3} \Rightarrow a_f \leq \frac{1}{\pi}\left(\frac{20}{169.8}\right)^2 = 0.0044 \, m$$

Crack is located in weld, hence use
$\Delta\sigma = \sigma_{max} - \sigma_{min}$ due to welding residual
stresses, SSH extract p 4:86
Dowling p 520 $\Rightarrow$

$$N = \frac{a_f^{1-\frac{m}{2}} - a_i^{1-\frac{m}{2}}}{C(F\Delta S\sqrt{\pi})^m (1-\frac{m}{2})} = \frac{(4.4 \cdot 10^{-3})^{-0.6} - (2 \cdot 10^{-4})^{-0.6}}{5 \cdot 10^{-13}(0.728(2 \cdot 97 \cdot \sqrt{\pi}))^{3.2} \cdot (-0.6)}$$

$N = 9.8 \cdot 10^6$ cycles

Check $\Delta K = 3.5 > \Delta K_{TH}$

Kontorsgrossen

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Q7 (12 p)

Thick-walled pressure vessel
(adjustable ends $\rightarrow \sigma_z = 0$)

$$\sigma_r(r) = A - B/r^2$$
$$\sigma_\varphi(r) = A + B/r^2$$

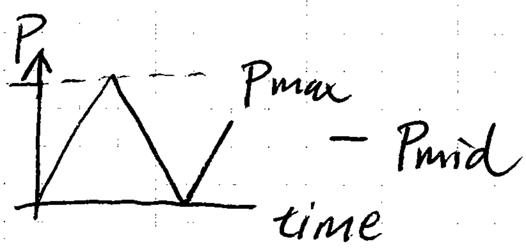
$$\left.\begin{array}{l}\sigma_r(d/2) = -p \\ \sigma_r(D/2) = 0\end{array}\right\} \Rightarrow \begin{array}{l} A = \frac{1}{3}p \\ B = \frac{4}{3}p\left(\frac{d}{2}\right)^2\end{array}$$

At inner surface we have $\sigma_r = -p$ $\sigma_\varphi = \frac{5}{3}p$

Follow handout Multi-axial Fatigue 2.11

$$\sigma_{ij}(t) = p(t)\begin{bmatrix}-1 & 0 & 0 \\ 0 & 5/3 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

Pulsating pressure $\Rightarrow$



$$\sigma_{ija} = \frac{P_{max}}{2}\begin{bmatrix}-1 & 0 & 0 \\ 0 & 5/3 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

$$\sigma_h(t) = \frac{1}{3}\sigma_{kk} = \frac{2}{9}p(t) \Rightarrow \sigma_{h,mid} = \frac{2}{18}P_{max}$$

The deviatoric stress tensor becomes

$$\sigma_{ij}^d(t) = p(t)\begin{bmatrix}-1-\frac{2}{9} & 0 & 0 \\ 0 & \frac{5}{3}-\frac{2}{9} & 0 \\ 0 & 0 & -\frac{2}{9}\end{bmatrix} = \frac{p(t)}{9}\begin{bmatrix}-11 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & -2\end{bmatrix}$$

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Q7 cont'd

Hence $\sigma^d_{ij,mid} = \frac{P_{max}}{18}\begin{bmatrix} -11 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

and the stress deviator amplitude

$$\sigma^d_{ij,a} = \frac{P_{max}}{18}\begin{bmatrix} -11 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

The Dang Van criterion, MF p 17

$$\sigma_{ERDV} = \frac{\sigma_{1,a} - \sigma_{3,a}}{2} + C_{DV}\,\sigma_{h,max}$$

$$\sigma_{ERDV} = \frac{P_{max}}{36}(13-(-11)) + C_{DV}\,\frac{2}{9}P_{max}$$

Determine C_{DV}, see MF p20-21

$\tau_{FL} = 275$ MPa $\Rightarrow$ $\frac{275-(-275)}{2} = \sigma_{eDV} \Rightarrow \sigma_{eDV} = 275$

$\sigma_{FLB} = 450$ MPa $\Rightarrow$

$$\frac{450-0}{2} + C_{DV}\,\frac{450}{3} = \sigma_{eDV}$$

$$\rightarrow C_{DV} = \frac{275-225}{150} = 0.333$$

Hence $\frac{P_{max}\cdot 24}{36} + 0.33\cdot\frac{2}{9}P_{max} \geq 275$

$$P_{max} \geq \frac{275}{0.74} = 372 \text{ MPa}$$

Kontorsgrossen