**Solutions to the written examination in *Fatigue Design* for:**
* IMP in Automotive Engineering
* IMP in Naval Architecture
* M4 MMA115 Utmattningsdimensionering

## THEORETICAL PART (14 p)

### Question 1 (4p)

a) The tensile overload will introduce a large plastic zone at the crack tip leading to compressive residual stresses in the vicinity of the (behind) crack tip. This reduces the effective stress intensity factor range for crack growth (a plasticity crack closure effect). An overload is detrimental e.g. if the crack is so long that the overload causes the stress intensity factor to exceed the fracture toughness. It is also detrimental if it induces the growth of a small crack that otherwise would be dormant.

b) When $R$ is increased the mean value of the loading is increased. Then the minimum $K$ is also increased, for constant $\Delta K$. This means that the closure effect are dimished giving a larger effective part of $\Delta K$ and hence a faster crack growth. So, the upper line, A, corresponds to a higher $R$-value.

### Question 2 (4 p)

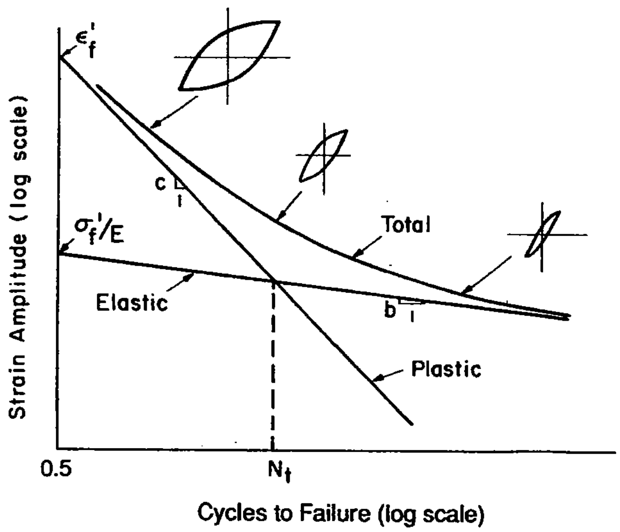
a) See the course material on multi-axial fatigue, p 9. One has

$$\sigma_1^d - \sigma_3^d = (\sigma_1 - \sigma_h) - (\sigma_3 - \sigma_h) = \sigma_1 - \sigma_3$$

b) See Dowling p 652-654.

$$\varepsilon_a = \sigma_f'/E\,(2N_f)^b + \varepsilon_f'\,(2N_f)^c$$

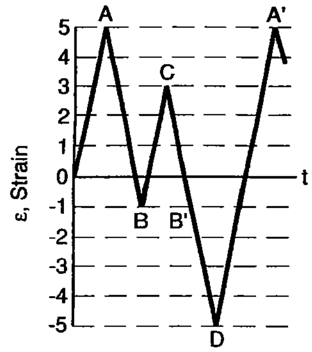
The Coffin-Manson relates the total strain amplitude $\varepsilon_a$ to the fatigue life $N_f$, the first term is the elastic strain amplitude and the second term is plastic strain amplitude. Tests are normally carried out with alternating, reversed loading, $R = -1$.



**THEORETICAL PART (14 P), cont'd**

**Question 3 (3 p)**

Rain flow count is described in Dowling p 404-406. For the sequence given, one has




One cycle B-C-B' with $\varepsilon_m = 1$ and $\varepsilon_a = 2$ and
One cycle A-D-A' with $\varepsilon_m = 0$ and $\varepsilon_a = 5$

**Question 4 (3 p)**

The Palmgren-Miner rule is described in Dowling p 402. The linear damage rule defines a cycle ratio $n/N$ where $n$ is the number of cycles at a stress level $S$ and $N$ is the fatigue life in cycles at the same stress level $S$. The damage fraction caused by the stress level $S_i$ is equal to $n_i/N_i$. The structure is exhausted, that is failure will occur when the sum of damage caused by all stress levels in the load spectrum equals unity that is $\Sigma_i\, n_i/N_i = 1$.

The Palmgren-Miner linear damage accumulation rule has one main shortcoming. It does not consider sequence effects, i e the load history is described by a load spectrum only. The theory predicts that the damage caused by a stress cycle is independent of when it occurs in the load history. This is contradicted by several experiments.

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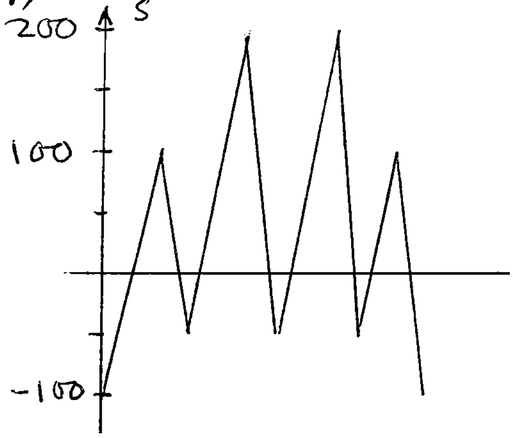
# Problem part (36 p)

**Q5 (12p)** TRANSFORMATION TO NOMINAL STRESS, ACCORDING TO FIG A8 (ALSO GIVING $K_t$) GIVES $S = \frac{P}{w_1 t} = \frac{P}{250}$, WITH P IN N.




A RAIN-FLOW COUNT GIVES

| | | | |
|---|---|---|---|
| I | 2 CYCLES | $S_a = 75$ MPa | $S_m = 25$ MPa |
| II | 1 CYCLE | $S_a = 125$ MPa | $S_m = 75$ MPa |
| III | 1 CYCLE | $S_a = 150$ MPa | $S_m = 50$ MPa |

SWT EQUIVALENT AMPLITUDES ARE

$$S_{a,eq} = \sqrt{S_a(S_a + S_m)}$$

| | | | |
|---|---|---|---|
| I | 2 CYCLES | WITH $S_{a,eq}$ = | 87 MPa |
| II | 1 –"– | –"– –"– | 158 MPa |
| III | 1 –"– | –"– –"– | 173 MPa |

WITH $w_2/w_1 = 60/50 = 1.2$ AND $g/w_1 = 0.1 \Rightarrow k_t = 1.9$

USING EG EQUATION 10.8

$$\alpha = 0.025\left(\frac{2070}{\sigma_u}\right)^{1.8} = 0.025\left(\frac{2070}{700}\right)^{1.8} \sim 0.17 \text{ mm}$$

AND $K_f = 1 + \frac{1}{1+\frac{\alpha}{g}}(k_t - 1) = 1 + \frac{1}{1+\frac{0.17}{5}}(1.9-1) \approx 1.87$

FROM THE EQUATIONS $\sigma_a = \sigma_f'(2N_f)^b$ AND $\sigma_a = k_f S_{a,eq}$

| | $S_{a,eq}$ (MPa) | $\sigma_a$ (MPa) | $N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b}$ |
|---|---|---|---|
| I | 87 | 163 | $(38\,000 \times 10^3)$ |
| II | 158 | 294 | $179 \times 10^3$ |
| III | 173 | 323 | $76 \times 10^3$ |

ONLY II AND III CONTRIBUTE SIGNIFICANTLY TO THE DAMAGE D

$$D = \left(\frac{1}{179} + \frac{1}{76}\right)10^{-3} = 0.0188 \cdot 10^{-3}$$

SO, THE NUMBER OF SEQUENCES IS

$$N = \frac{1}{D} = 53 \cdot 10^3$$

CHALMERSFASTIGHETER

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Q6 (12p)

a) Fracture mechanics

$K_I = F \cdot \sigma \cdot \sqrt{\pi a}$ ; $\Delta K = F \Delta\sigma \sqrt{\pi a}$

With crack in weld with residual stresses (tensile) present we have

$\Delta\sigma = \sigma_{max} - \sigma_{min} = 150$ MPa

Through-the-thickness edge crack

Dowling p 301 Case (c) $a_i/b \approx 0 \Rightarrow F = 1.12 \approx$ const.

Static fracture $K_{max} = \sigma_{max} F \sqrt{\pi a_c} = K_{Ic}$

$$\Rightarrow a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{F\sigma_{max}}\right)^2 = \frac{1}{\pi}\left(\frac{40}{1.12 \cdot 75}\right)^2 = 0.072 \text{ m}$$

For $F \approx$ constant, number of cycles to failure is obtained from Dowling p 520

$$N = \frac{a_c^{1-\frac{m}{2}} - a_i^{1-\frac{m}{2}}}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m \left(1-\frac{m}{2}\right)}$$

using Paris law $\frac{da}{dN} = C(\Delta K_I)^m$

$$\text{Hence } N = \frac{0.072^{-\frac{1}{2}} - 0.0015^{-\frac{1}{2}}}{1 \cdot 10^{-12}\left(1.12 \cdot 150\sqrt{\pi}\right)^3 \cdot \left(-\frac{1}{2}\right)} \approx 1.67 \cdot 10^6 \text{ cycles}$$

$\boxed{\text{N insensitive to } a_c \text{ value}}$

Note: $\sigma_{max}$ is probably higher $\approx \sigma_Y \rightarrow a_c$ lower $\rightarrow N$ lower

Kontorsgrossen

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Q6 (12p) cont'd

b) Design codes for welds
Use hand-out from Sheet Steel Handbook

Design criterion, p 4:76 & p 4:73

$$\sigma_r \cdot \gamma_f \leq \frac{f_{rk} \cdot \varphi_m \cdot \varphi_t \cdot \varphi_e}{\gamma_{mn}}$$

where the characteristic fatigue resistance is $f_{rk} = C \left( \frac{2 \cdot 10^6}{n_t} \right)^{1/3}$

Joint Class: Double-V, quality WB
loaded $\perp$ weld line $\rightarrow$
Case 10 $C = 90$

Risk of failure, p 4:76 $Q_B = 0.5 \rightarrow \gamma_{mn} = 0.75$
weld, p 4:74 $\rightarrow \varphi_m = 1$
thickness $t = 10$ mm, p 4:74 $\rightarrow \varphi_t = 1.04$
weld residual stress unknown, p 4:74 $\rightarrow \varphi_e = 1$
and $\gamma_f = 1 \implies$

$$\sigma_r \leq \frac{C \cdot \varphi_t}{\gamma_{mn}} \left( \frac{2 \cdot 10^6}{n_t} \right)^{1/3} \rightarrow$$

$$n_t \leq 2 \cdot 10^6 \left( \frac{C \cdot \varphi_t}{\sigma_r \cdot \gamma_{mn}} \right)^3 \leq 2 \cdot 10^6 \left( \frac{90 \cdot 1.04}{150 \cdot 0.75} \right)^3 \simeq 1.2 \cdot 10^6$$

cycles

**Question 7 (12 p)**

See the course material on Multi-axial fatigue on p 12-13 and 16-17

We get the material parameters $\sigma_{\text{eDV}} = \tau_{\text{FL}}$ and $c_{\text{DV}} = 3(\tau_{\text{FL}} / \sigma_{\text{FL}} - 1/2) = 3/18$.
We get the normal stress amplitude as $\sigma_{\text{a}} = 4P_{\text{a}} / \pi d^2 = 95$ MPa. The static shear stress will not influence the fatigue impact according to Dang Van.

The Dang Van stress thus becomes

$$\sigma_{\text{EQ,DV}} = \frac{\sigma_{1,\text{a}} - \sigma_{2,\text{a}}}{2} + c_{\text{DV}}\sigma_{\text{h,max}} = \frac{\sigma_{\text{a}}}{2} + \frac{3 \cdot \sigma_{\text{a}}}{18 \cdot 3} = \frac{5\sigma_{\text{a}}}{9} = \frac{5 \cdot 95}{9} = 53 \text{ MPa}$$

The safety factor becomes
$SF = \sigma_{\text{eDV}} / \sigma_{\text{EQ,DV}} = 100/53 = 1.89$