# Written examination in *Fatigue Design* for:

* IMP in Automotive Engineering
* IMP in Naval Architecture
* M4 MMA115 Utmattningsdimensionering

- Date: Tuesday 2007-08-28
- Time: $14^{00} – 18^{00}$
- Location: V – Building
- Teacher: Lennart Josefson, phone 7721507
- Solutions will be available 2007-08-29 at $08^{30}$ at the Department of Applied Mechanics
- Corrected examinations will be available 2007-09-19 at the Department of Applied Mechanics

The written examination consists of two parts: one theoretical part of 14 points in total, and one problem solving part consisting of three problems each with a value of 12 points. A fully correct solution of the complete exam gives a maximum of 50 points.

*Theoretical part:*
This part must be solved first. No aids are allowed, ie this part is to be solved without textbooks, hand calculator etc. The solution of this part shall be handed in separately, before you continue with the problem part.

*Problem part:*
This part may be solved using text books and text material distributed during the course, see below. These aids may be collected when the solutions to the theoretical part have been handed in. Short notes in the course material are allowed. Note, however, that solved problems to exercises from the course are NOT allowed as aids. A hand calculator may be used. When in doubt about the hand calculator being a full computer, the teacher should be consulted to permit the hand calculator.
**NOTE: To obtain maximum points for each problem, the solution must be clearly motivated and all the equations used from the literature should have a clear reference.**

Allowed literature during the problem part:

- Dowling, N.E. (2006) *Mechanical Behavior of Materials*, Third Edition, Prentice-Hall, Upper Saddle River, NJ, USA, 912 pp.
- Fatigue Design (MMA115) Course Material on Multiaxial Fatigue
- Extract from SSAB Sheet Steel Handbook
- Mathematical Tables and Handbooks in Engineering like KTH:s *Formelsamling i hållfasthetslära*, or Roark R.J. and Young, W.C., *Formulas for Stress and Strain*.

The marks are given as (the sum of points from the theoretical and problem parts):

- Not passed 0 – 19 points
- 3 = passed 20 – 29 points
- 4 = very good 30 – 39 points
- 5 = excellent 40 - 50 points

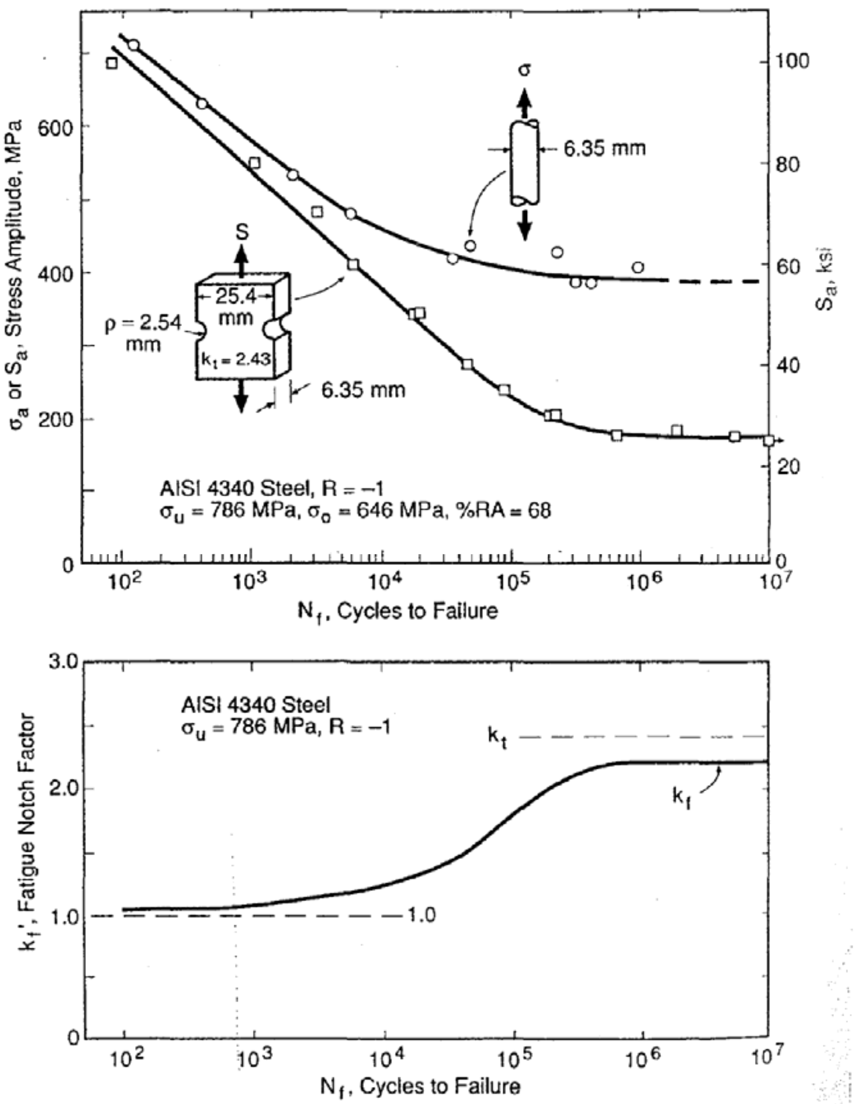
**THEORETICAL PART (14 p)**

**Question 1 (4p)**

Explain why $\Delta K$ and $R$ are relevant parameters for the modelling of fatigue crack growth.

**Question 2 (3p)**

The Figure shows the variation of the fatigue notch factor $k_f$' with life. Explain this behaviour qualitatively both for long and short lives.




**Question 3 (4p)**

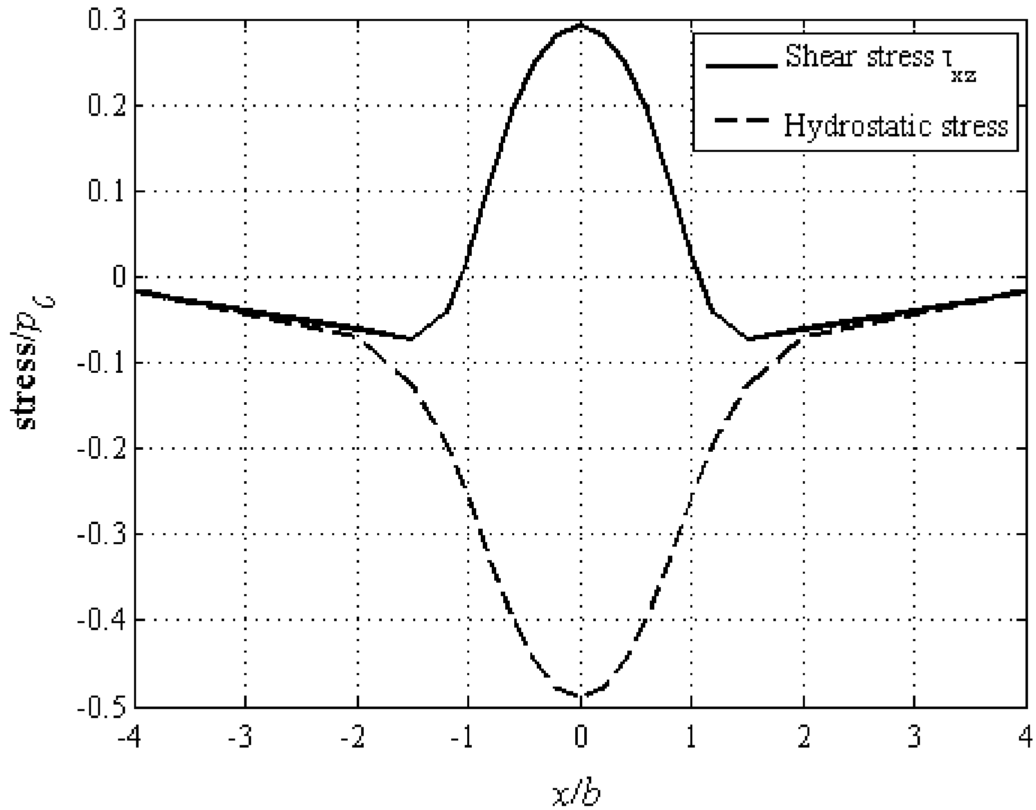
Sketch the typical experimental result of the influence of the mean stress $\sigma_m$ in a fatigue test, that is draw a $\sigma_m$ - $\sigma_a$ plot. Give two relations for the equivalent completely reversed stress amplitude, $\sigma_{ar}$ used for life estimates in stress based HCF design, based on

- the modified Goodman equation
- The Smith-Watson-Topper (SWT) relation

**THEORETICAL PART (14 p), cont'd**

**Question 4 (3p)**

A steel cylinder is rolling on a steel plate with no interfacial friction. For this case the largest Dang Van equivalent stress will occur below the surface[1]. The figure shows the evolution of the state of stress at a material point below the surface. The shear stress, $\tau_{xz}$, on a shear plane inclined 45° to the surface and the hydrostatic stress are plotted as a function of the normalized distance to the centre of the contact from the material point studied. In other words, the plot shows the stress evolution during a load passage, which is equal to one load cycle. The stresses are normalized by the peak contact pressure $p_0$. Determine the maximum Dang Van stress $\sigma_{\mathrm{EQDV}}$ for the studied material point and shear plane. Express $\sigma_{\mathrm{EQDV}}$ as a function of the peak contact pressure $p_0$. Consider the cases when the material parameter in the Dang Van criterion are $c_{\mathrm{DV}} = 0$ and $c_{\mathrm{DV}} = 1$, respectively. Hint: A state of plane strain exists, which for instance means that $\tau_{xy}$ and $\tau_{yz}$ are zero.

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[1] The distance below the surface will depend on the size of the contact patch, *b*, but that is nothing you need to worry about here. The stress magnitudes in the task are evaluated at this critical distance.

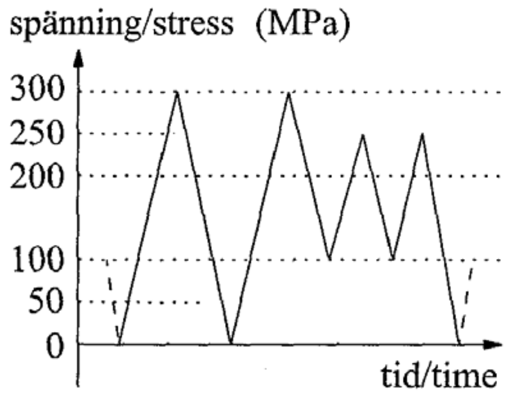
**PROBLEM PART (36 p)**

**NOTE: To obtain maximum points for each problem, the solution must be clearly motivated and all the equations used from the literature should have a clear reference (author, page and equation number)**

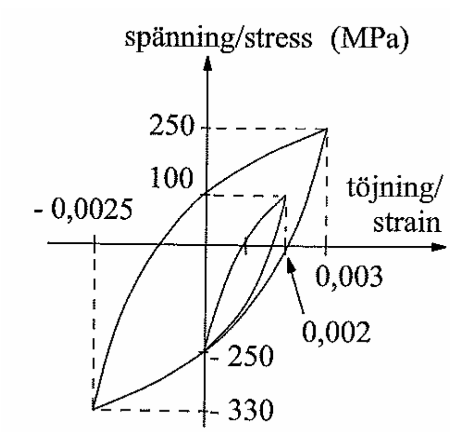
**Question 5 (12p)**
A large plate, loaded in uni-axial tension, is subjected to a load sequence shown in the Figure. This load sequence is repeated. The material has a *S-N* Curve given by

$\sigma_a = 430 - 55\log(N)$     [MPa]

Determine the number of sequences to fatigue failure, based on the Palmgren-Miner damage accumulation rule. The influence of mean stress may be neglected.




**Question 6 (12p)**
The diagram shows a stabilized hysteresis loop for a material subjected to a load sequence of two cycles. The two cycles give a stress-strain curve according to the Figure. Use the modified Morrow relationship to determine the number of cycles to fatigue failure. Use the material data: $E$ = 200 GPa, $\sigma_f'$ = 900 MPa, $\varepsilon_f'$ = 0.26, $b$ = -0.095, $c$ = -0.47.



**PROBLEM PART (36 p), cont'd**

**Question 7 (12p)**

An attachment is welded to a load-carrying box beam in a truck. The beam is subject to a membrane stress as shown in the Figure. The risk of failure is $Q_B = 0.5$. Both the attachment and the beam has the plate thickness $t = 8$ mm. You may put $\gamma_f = 1$.

a) What joint class value, *C*-value, and thus weld quality level, *WA, WB* or *WC*, is needed for the beam to have a fatigue life of $n_t = 1*10^7$ cycles if it is subject to a variable amplitude load with the dimensioning stress range $\sigma_{rd}$ =120 MPa and the collective parameter $\kappa = 1/3$?

b) Using the joint class value *C* from a) above, determine the fatigue life if the beam instead is subject to a constant amplitude stress range $\sigma_{rd}$ =100 MPa.