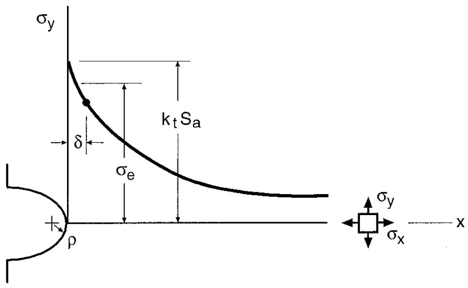
Solutions to the written examination in *Fatigue Design* 2008-05-27

for the Master's programs Applied Mechanics (MPAME),
Advanced Engineering Materials (MPAEM) and Naval Architecture (MPNAV)

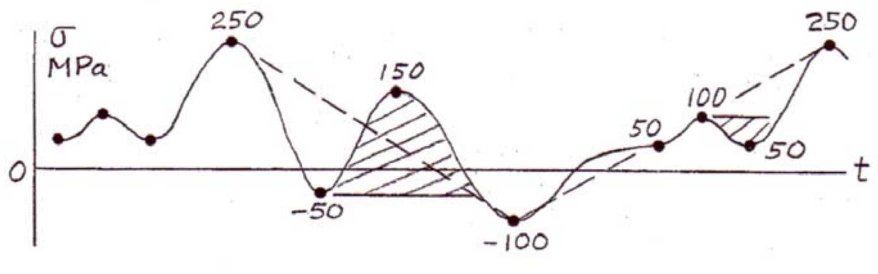
THEORETICAL PART (14 p)

Question 1 (4p)

a) See Dowling p 468 – 471. At long lives the conditions at the notch root are elastic. Then the nominal stress multiplied by the stress concentration factor k_t should, ideally, be equal to the fatigue limit. However, this maximum stress occurs only at a point and decreases rapidly. It is believed that stresses material is not sensitive to peak stresses, but rather to the average stress that acts over a small, finite region (comparable to several grain diameters). Hence, the effective concentration factor, the fatigue notch factor, $k_f < k_t$, For large notch radii ρ values the stresses decrease slower and hence $k_f \approx k_t$.



b)
Use Dowling p 446-448. Inserting stress values we identify two local and one global cycle



Cycle	Mid stress (MPa)	Amplitude stress (MPa)
-50 to 150 to -100 local	50	100
100 to 50 to 250 local	75	25
250 to - 100 to 250 global	75	175

THEORETICAL PART (14 p), cont'd

Question 2 (4p)

a)

A See Dowling p 738-739. The critical plane concept is an approach used in numerical calculations to find the material plane where the largest fatigue damage, according to the fatigue criterion, occurs during cyclic loading. In a multi-axial fatigue situation, the principal stress and strain directions vary in time. The plane that experiences the largest fatigue damage is identified as the "critical plane". Fatigue crack initiation is believed to be caused by the cyclic shear strain acting in the critical plane (forming slip bands) which may be enhanced by stresses normal to the critical plane.

b)

In the welding process, the material is rapidly heated to a high temperature followed by rapid cooling leading to local yielding first in compression then during cooling in tension. The material will therefore as a result contain residual stresses with high tensile magnitudes at the weld (of the yield stress level, σ_{yield}) balanced by compressive stresses away from the weld region. The material at the weld toe (in a fillet weld) will be subject to the sum of a cyclic external stress (with stress range $\Delta\sigma$) and a high tensile mean stress. Conservatively, one may assume that the material at the weld toe is subject to a stress that varies between σ_{yield} and $\sigma_{\text{yield}} - \Delta\sigma$ irrespective of the stress ratio of the external stress. Moreover, the welding process will in many cases create geometric discontinuities that will initiate macroscopic cracks (cracks with lengths 0.1 - 0.5 mm).

Question 3 (3p)

a)

See Dowling p 683-684. Consider a plate, made of an elastoplastic material, containing a notch. The plate is loaded with a nominal stress S, the nominal strain being $e = S/E$.
At the bottom of the notch the maximum local stress and strain is σ and ε respectively.
The elastic stress concentration factor is $k_t = \sigma/S$.

At the notch (for an elasto-plastic behaviour) we may define the stress and strain concentration factors as $k_\sigma = \sigma/S$ and $k_\varepsilon = \varepsilon / e$.
Neuber's rule states that

$$k_t = \sqrt{k_\sigma k_\varepsilon}$$

Using the nominal strain e above one obtains

$$\sigma\varepsilon = \frac{(k_t S)^2}{E}$$

For a given load, material and geometry, the product of the local max stress and strain is constant. Since the stress and strain is related by the elasto-plastic stress-strain curve a solution can be obtained for their values.

THEORETICAL PART (14 p), cont'd

Question 3 b)

See Dowling p 728-733: In the strain-based approach, the Coffin-Manson equation

$$\varepsilon_a = \sigma_f'/E\,(2N_f)^b + \varepsilon_f'\,(2N_f)^c$$

relates the total strain amplitude ε_a to the fatigue life N_f, the first term is the elastic strain amplitude and the second term is plastic strain amplitude. The mean stress σ_m is included in the first elastic term, the modified Morrow approach, as

$$\varepsilon_a = \sigma_f'/E\,(1 - (\sigma_m/\sigma_f'))(2N_f)^b + \varepsilon_f'\,(2N_f)^c$$

In the Smith-Watson-Topper approach we have

$$\sigma_{max}\varepsilon_a = (\sigma_f')^2/E\,(2N_f)^{2b} + \sigma_f'\varepsilon_f'\,(2N_f)^{b+c}$$

Question 4 (3p)

a)
Plastic yield is quantified e.g. by the von Mises and Tresca effective stresses. Multiaxial HCF impact is quantified e.g. by the Crossland and Dang Van criteria. A static shear stress will increase the von Mises and Tresca effective stresses, but not the Crossland and Dang Van equivalent stresses. In physical terms this corresponds to the fact that a static shear stress may introduce plastic deformation, which will shift the response from high cycle fatigue (globally elastic response) to LCF (globally inelastic response).

b)

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{(\sigma_1 - \sigma_h) - (\sigma_2 - \sigma_h)}{2} = \frac{\sigma_1^d - \sigma_2^d}{2}$$

PROBLEM PART (36 p)

NOTE: To obtain maximum points for each problem, the solution must be clearly motivated and all the equations used from the literature should have a clear reference (author, page and equation number)

SP *Fatigue Design 080527*

Q5

RF COUNTING GIVES

$i=1$ 1 CYCLE 3 ± 7 kN

2 1 CYCLE 5 ± 5 kN

3 19 CYCLES 4 ± 4 kN

GIVING EQUIVALENT PURE AMPLITUDE LOADS

$$P_{aeq1} = \sqrt{7 \cdot 10} = 8.37 \text{ kN}$$
$$P_{aeq2} = \sqrt{5 \cdot 10} = 7.07 \text{ kN}$$
$$P_{aeq3} = \sqrt{4 \cdot 8} = 5.66 \text{ kN}$$

THE RELATIONSHIP

$$P = 40 N^{b}$$

FOR MAXIMUM LOADS HAS TO BE CHANGED TO THE CORRESPONDING AMPLITUDE RELATIONSHIP BY

$$P_{aeq} = \sqrt{P_{max} P_a} = \sqrt{P \frac{P}{2}} = \frac{1}{\sqrt{2}} P$$

I.E.

$$P_{aeq} = \frac{40}{\sqrt{2}} N^{b} = 28.3 N^{-0.13}$$

OR

$$\log P_{aeq} = \log 28.3 - 0.13 \log N$$

THE LIVES FOR THE THREE TYPES OF CYCLES ARE

$$\log N_1 = \frac{\log 28.3 - \log 8.37}{0.13} \Rightarrow N_1 = 11760$$
$$\log N_2 = \frac{\log 28.3 - \log 7.07}{0.13} \Rightarrow N_2 = 42800$$
$$\log N_3 = \frac{\log 28.3 - \log 5.66}{0.13} \Rightarrow N_3 = 236900$$

PROBLEM PART (36 p) cont'd

Q5 cont'd NOW THE DAMAGE FOR ONE SEQUENCE IS

$$D = \sum_{i=1}^{3} \frac{n_i}{N_i} = \frac{1}{11700} + \frac{1}{42800} + \frac{19}{236900} = 1.89 \cdot 10^{-4}$$

NUMBER OF SEQUENCES

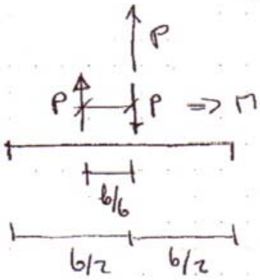
$$\frac{1}{D} = 5.3 \cdot 10^{3}$$

Q6 Dowling p553

$C_0 = 5.11 \cdot 10^{-10}$

$m = 3.24$

$\gamma = 0.42 \quad (R > 0)$



$\Rightarrow M = \frac{Pb}{6}$

THE LOAD CAN BE DIVIDED IN SYMMETRIC TENSION AND A BENDING MOMENT AND K_I CAN THEN BE OBTAINED FROM Fig 8.12 AND 8.13, ASSUMING THAT $a/b < 0.13$, AS: (Dowling p 344-345)

$$K_I = 1.12 \frac{P}{bt}\sqrt{\pi a} + 1.12 \frac{6Pb}{6b^2 t}\sqrt{\pi a} = 2 \cdot 1.12 \frac{P}{bt}\sqrt{\pi a}$$

$$K_{I,max} = K_{Ic}/4 = \frac{130}{4} = 32.5 = 2 \cdot 1.12 \cdot \frac{80000}{80 \cdot 12}\sqrt{\pi a_{max}} \Rightarrow$$

$a_{max} = 0.00965$ m. $\left(\frac{a_{max}}{b} = \frac{9.65}{80} < 0.13 \right)$

$a_{init} = 0.005$ m.

$$\Delta K = 2.24 \cdot \frac{60000}{80 \cdot 12}\sqrt{\pi a} = 140\sqrt{\pi a}$$

$$R = \frac{20000}{80000} = 0.25$$

$$\frac{da}{dN} = C(\Delta K)^m \quad \text{with } C = \frac{C_0}{(1-R)^{m(1-\gamma)}} = \frac{5.11 \cdot 10^{-10}}{(0.75)^{3.24 \cdot 0.58}} =$$

$$= 8.77 \cdot 10^{-10}$$

Now

$$N = \frac{a_{max}^{1-\frac{m}{2}} - a_{init}^{1-\frac{m}{2}}}{\left(1-\frac{m}{2}\right) C \left(140\sqrt{\pi}\right)^m}$$

PROBLEM PART (36 p) cont'd

SOLUTION

Q6 (CONT)

$$N = \frac{(0{,}00965)^{-0{,}62} - (0{,}005)^{-0{,}62}}{-0{,}62 \cdot 8{,}77 \cdot 10^{-13} (140\sqrt{\pi})^{3{,}24}} =$$

$$= \frac{8{,}94}{0{,}62 \cdot 8{,}77 \cdot 10^{-13} \cdot 5{,}74 \cdot 10^{7}} = 0{,}286 \cdot 10^{6}$$

$\therefore$ 286 000 CYCLES.

Check if growth occurs; $\Delta K_I \geq \Delta K_{Th}$

Dowling p 556 →

$$\Delta K_{Th} = 7.0(1 - 0{,}85R) \quad [MPa\sqrt{m}]$$

$$R = 0{,}25 \rightarrow \Delta K_{Th} = 5{,}5$$

Initially $\Delta K_I = 17.5$ ($a_{init} = 0.005\,m$) → OK

Q7

All stresses in MPa, $\sin(\omega t)$ is omitted.

a)

Material parameters, Multi-axial fatigue p 21:

Alternating tension: $\frac{250-0}{2} + c_{DV}\frac{250+0+0}{3} = \sigma_{eDV}$ and

Pulsating tension: $\frac{200-0}{2} + c_{DV}\frac{400+0+0}{3} = \sigma_{eDV}$

gives $c_{DV} = 1/2$ and $\sigma_{eDV} = 167$ MPa.

Use Multi-axial Fatigue, p 13

$\tau_{EQ,DV} = \tau_{Tresca,\,a} + c_{DV} + \sigma_h$ where $= (\sigma_{1,a} - \sigma_{3,a}) / 2$

PROBLEM PART (36 p) cont'd

Hence, principal stress amplitudes are calculated from

$$\sigma_{1,2} = \frac{150+50}{2} \pm \sqrt{\left(\frac{150-50}{2}\right)^2 + 150^2} = 100 \pm 158, \text{ which gives}$$

$$\sigma_{1,2,3} = 258, 0, -58 \left(\cdot \sin(\omega t)\right)$$

The Dang Van stress is then

$$\tau_{\mathrm{EQ,DV}} = \frac{258+58}{2} + \frac{1}{2} \cdot \frac{258-58+0}{3} = 191 > \sigma_{\mathrm{eDV}}$$

Hence we will have fatigue initiation.

b)
Critical crack length based on LEFM:
The crack will grow perpendicular to the largest principal stress. A relatively small crack in large steel plate gives, Dowling p 327,
$K_{\mathrm{I}} = S_{\mathrm{g}}\sqrt{\pi a}$ with $S_{\mathrm{g}} = \sigma_1 = 258 \cdot \sin(\omega t)$.

Fracture occurs at $\max(S_{\mathrm{g}})$, i.e. $\sin(\omega t) = 1$, when $K_{\mathrm{I}} = K_{\mathrm{Ic}}$ (Dowling p 317):

$$K_{\mathrm{I}} = 258\sqrt{\pi a_{\mathrm{c}}} = 100 = K_{\mathrm{Ic}} \Leftrightarrow a_{\mathrm{c}} = \left(\frac{100}{258\sqrt{\pi}}\right)^2 = 4.8 \text{ cm.}$$