Solutions to the written examination in *Fatigue Design* 2009-05-25

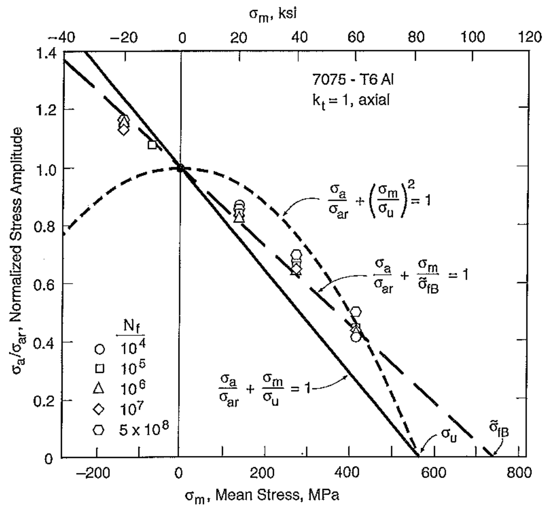
for the Master's programs Applied Mechanics (MPAME),
Advanced Engineering Materials (MPAEM) and Naval Architecture (MPNAV)

THEORETICAL PART (14 p)

Question 1 (4p)

a)
We have Log $N_{97.8}$ = Log N_{50} - 2σ = Log $(6\cdot10^4)$ - 2σ = 4.78 - 2·0.2 = 4.38
Hence $N_{97.8} \approx 2.5\cdot10^4$.

b)

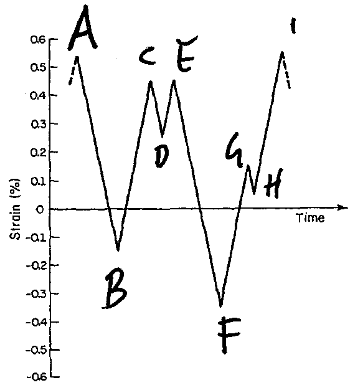


See Dowling p 429. The equivalent completely reversed stress amplitude σ_{ar} is defined on Dowling p 431. We have for the Modified Goodman equation (9.17) and Smith-Watson-Topper (SWT), (9.18) respectively

$$\sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma'_f}}$$ where σ'_f is a material constant from the Basquin equation (*S-N* curve)

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$$, where $\sigma_{max} = \sigma_m + \sigma_a$

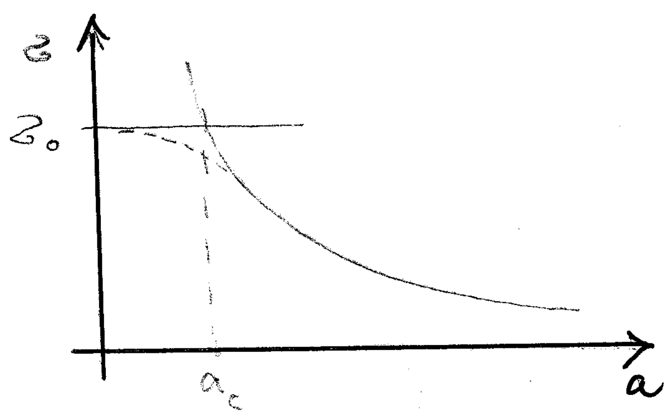
Question 2 (3p)



Use Dowling p 446-447. Inserting strain values we identify cycles as:

Cycle	Mid strain (%)	Amplitude strain (%)
C-D-E	0.35	0.10
B-E-F	0.15	0.30
G-H-I	0.10	0.05
A-F-I	0.10	0.45

Question 3 (3p)



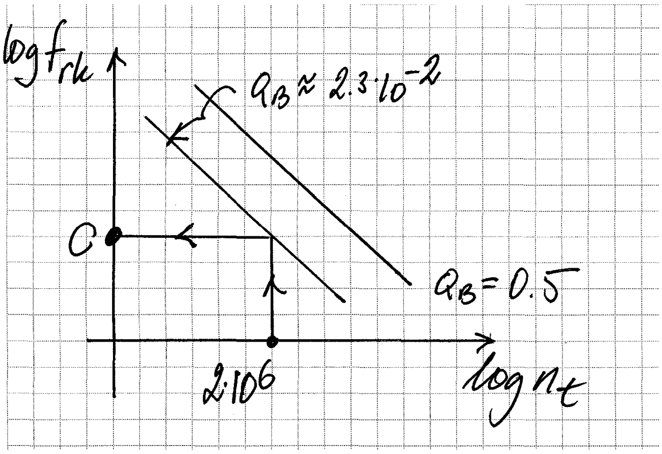
Brittle fracture occurs when $K_{IC} = \sigma 1.12\sqrt{\pi a}$ For small cracks ductile tearing occurs when $\sigma = \sigma_o$. An approximate crack length marking the change from brittle to ductile fracture is obtained from $K_{IC} = \sigma_o 1.12\sqrt{\pi a_c}$

Hence $a_c = (K_{IC}/(1.12 \cdot \sigma_o))^2/\pi \approx (120/(1.12 \cdot 620))^2/3.14 \approx 0.01$ m = 10 mm

THEORETICAL PART (14 p), cont'd

Question 4 (4p)

a)



The joint class C is the stress range for which the welded joint can sustain the fatigue life $n_t = 2{\cdot}10^6$ cycles with the risk of failure $Q_B = 0.023$ (2 standard deviations shift of the median curve for the welded joint).

b)

See the hand-out on Multi-axial fatigue p 16-17. Initiation of fatigue (cracks) will take place (for proportional loading with fixed principal stress directions) if

Dang Van: $\sigma_{EQDV} \geq \sigma_{eDV}$ where $\sigma_{EQDV} = (\sigma_{1a} - \sigma_{3a})/2 + c_{DV}\, \sigma_{h,max}$
Here the first term corresponds to τ_a, the maximum shear stress amplitude in a critical plane, σ_h = is the hydrostatic stress and c_{DV} and σ_{eDV} are material parameters determined from two different fatigue limits.

Crossland: $\sigma_{EQC} \geq \sigma_{eC}$ where $\sigma_{EQDV} = \max_t (\sigma_{v,Ma}) + c_C\, \sigma_{h,\,max}$
Here the first is the von Mises effective stress and c_C and σ_{eC} are material parameters determined from two different fatigue limits.

PROBLEM PART (36 p)

NOTE: To obtain maximum points for each problem, the solution must be clearly motivated and all the equations used from the literature should have a clear reference (author , page and equation number)

Q5 For the plate we have

$$\left.\begin{array}{l}\frac{W_2}{W_1} = 1.2 \\ \frac{s}{W_1} = \frac{1}{10}\end{array}\right\} \Rightarrow k_t = 2.5$$

The strain amplitude during cyclic loading $S_a = \pm 200$ MPa is obtained from Neubers rule, Dowling p 686

For perfectly plastic material

$$\varepsilon_a = \frac{(k_t S)^2}{\sigma_0 E} = \frac{(2.5 \cdot 200)^2}{400 \cdot 2 \cdot 10^5} = 0{,}0031$$

The Coffin-Manson relation, Dowling p 721

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \longrightarrow$$

$$0.0031 = \frac{1200}{2 \cdot 10^5}(2N_f)^{-0.1} + 1.0(2 \cdot N_f)^{-0.62} \longrightarrow$$

$N_f \approx 3 \cdot 10^4$ cycles

PROBLEM PART (36 p) cont'd

Question 6
Machined surface use Dowling p 479, and 1 ksi ≈ 6.9 MPa, hence

Machined $m_s \approx 0.72$ gives $\sigma_{FL} = \pm 360$ and $\sigma_{FLP} = 425 \pm 319$ MPa (or $\sigma_{FLP} = 319 \pm 319$)

$$\sigma_{EQDV} = \frac{360-0}{2} + c_{DV}\frac{360}{3} = \sigma_{eDV} = \frac{319-0}{2} + c_{DV}\frac{744}{3}$$
(or $\sigma_{EQDV} = \frac{360-0}{2} + c_{DV}\frac{360}{3} = \sigma_{eDV} = \frac{319-0}{2} + c_{DV}\frac{638}{3}$)

$c_{DV} = 0.16$ (or $c_{DV} = 0.22$) and $\sigma_{EQDV} \approx 199$ MPa (or $\sigma_{EQDV} \approx 206$ MPa)

For the biaxial loading, use Ekberg Multi-axial fatigue p 17 with σ_x and σ_y being principal stresses.

$$\sigma_{EQDV} = \frac{150+150}{2} + c_{DV} \cdot 0 = 150 \text{ MPa}$$

$SF_{DV} = \frac{199}{150} = 1.33$ (or $SF_{DV} = \frac{206}{150} = 1.37$)

$\sigma_{x,res} = 100 \Rightarrow \sigma_{h,res} = \frac{100}{3} \Rightarrow \sigma_{EQDV} = 150 + 0.16 \cdot \frac{100}{3} \approx 155$ (or $\sigma_{EQDV} = 150 + 0.22 \cdot \frac{100}{3} \approx 157$)

$SF_{DV} = \frac{199}{155} = 1.28$ (or $SF_{DV} = \frac{206}{157} = 1.31$)

PROBLEM PART (36 p) cont'd

Note change AISI4030 to AISI4340, Dowling p 553

Q7 AISI 4030 : $\sigma_0 = 1255$ MPa, $K_{Ic} = 130$ MPa$\sqrt{m}$, $C_0 = 5{,}11 \cdot 10^{-13}$

$m = 3.24$ AND $\gamma = 0$ FOR $R < 0$.

$$S_A = \frac{32M_A}{\pi d^3} = \frac{32PL}{\pi d^3}$$

$$S_B = \frac{32M_B}{\pi D^3} = \frac{32 \cdot 2PL}{\pi\left(\frac{4}{3}d\right)^3} = \frac{32PL}{\pi d^3}\left(\frac{2 \cdot 27}{64}\right) = \frac{32PL}{\pi d^3}\left(\frac{54}{64}\right) < S_A$$

IE A IS THE CRACK SITE TO BE EVALUATED.

$$S_A = \frac{32 \cdot 16000 \cdot 200}{\pi \cdot 60^3} = 151 \text{ MPa}$$

FOR THIS CASE Fig 8.17 IN DOWLING GIVES

$K_I = 0.73\, S\sqrt{\pi a}$ (IF $a < 0.35d$)

WITH $a_0 = 0.003$ m $K_{I0} = 0.73 \cdot 151\sqrt{\pi\, 0.003} = 10.7$ MPa$\sqrt{m}$

THE MAXIMUM K_I ALLOWED IS $K_{Ic}/5 = 26$ MPa$\sqrt{m}$

GIVING $a_{max} = \frac{1}{\pi}\left(\frac{26}{0.73 \cdot 151}\right)^2 = 0.0177$ m ($< 0.35d$)

$a > 2.5\left(\frac{K_I}{\sigma_0}\right)^2$ FULFILLED, IE LEFM.

FOR ROTATING BENDING $R = -1$ SO $\gamma = 0$, MEANING THAT

$\overline{\Delta K_I} = \frac{\Delta K_I}{2} = K_{I,max}$. (OR $\overline{\Delta S} = S_{MAX}$)

THEN

$$N = \frac{a_{max}^{1-\frac{m}{2}} - a_0^{1-\frac{m}{2}}}{C_0\left(0.73 \cdot 151\sqrt{\pi}\right)^m\left(1-\frac{m}{2}\right)} = \frac{0.0177^{-0.62} - 0.003^{-0.62}}{5{,}11 \cdot 10^{-13}\left(0.73 \cdot 151\sqrt{\pi}\right)^{3.24}(-0.62)}$$

$= 2.9 \cdot 10^6$ CYCLES.