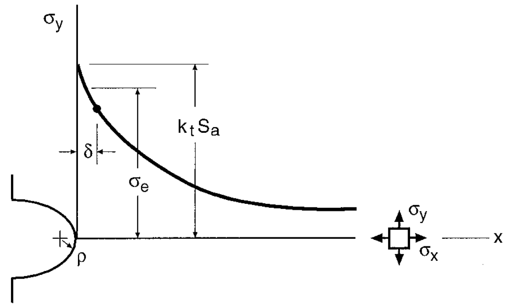
Solutions to the written examination in Fatigue Design 2010-05-24

(MMA 115) for the Master's programs Applied Mechanics (MPAME), Advanced Engineering Materials (MPAEM) and Naval Architecture (MPNAV)

THEORETICAL PART (14 p)

Question 1 (4p)

See Dowling p 468 – 471. At long lives the conditions at the notch root are elastic. Then the nominal stress multiplied by the stress concentration factor k_t should, ideally, be equal to the fatigue limit. However, this maximum stress occurs only at a point and decreases rapidly. It is believed that stresses material is not sensitive to peak stresses, but rather to the average stress that acts over a small, finite region (comparable to several grain diameters). Hence, the effective concentration factor, the fatigue notch factor, $k_f < k_t$, For large notch radii ρ values the stresses decrease slower and hence $k_f \approx k_t$.



Question 2 (4p)

a) See Dowling p 641-642. The cyclic stress-strain curve is determined from tests with prescribed strain amplitude at R_ε = -1. Stable loops at half fatigue life and for different strain amplitudes are used as shown in the figure below. A line from the origin through the tips of the loops (O-A-B-C) forms the cyclic stress strain curve. It is of the Ramberg-Osgood form:

$$\varepsilon_a = \sigma_a/E + (\sigma_a/H')^{1/n'}$$

THEORETICAL PART (14 p), cont'd

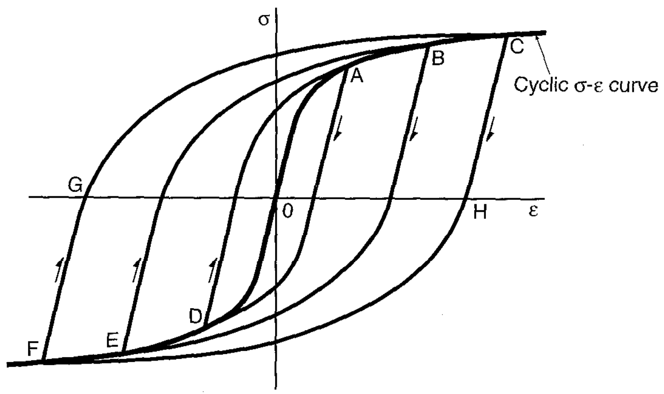


Figure 12.19 Cyclic stress–strain curve defined as the locus of tips of hysteresis loops. Three loops are shown, *A-D*, *B-E*, and *C-F*. The tensile branch of the cyclic stress–strain curve is *O-A-B-C*, and the compressive branch is *O-D-E-F*.

b) See Dowling p 728-733: In the strain-based approach, the Coffin-Manson equation

$$\varepsilon_a = \sigma_f'/E\,(2N_f)^b + \varepsilon_f'\,(2N_f)^c$$

relates the total strain amplitude ε_a to the fatigue life N_f, the first term is the elastic strain amplitude and the second term is plastic strain amplitude. In the modified Morrow approach, the mean stress σ_m is included in the first elastic term,

$$\varepsilon_a = \sigma_f'/E\,(1 - (\sigma_m/\sigma_f'))(2N_f)^b + \varepsilon_f'\,(2N_f)^c$$

Question 3 (4p)

a) See Dowling p 322-326. The singularity is a square root singularity, i.e. the stress varies with $1/\sqrt{r}$, where r is the distance from the crack tip. The stress close to the crack tip is proportional to the stress intensity factor, i.e. $\sigma \sim K_I/\sqrt{r}$. The validity of LEFM is related to the size of the plastic zone. If the plastic zone is large, it redistributes the stress at the crack tip and the singularity diminishes and K_I is not a meaningful measure of the stress at the crack tip any more.

b) The reasoning is that the stress magnitude at the crack tip is reduced from (theoretically, according to linear elastic theory) infinity to roughly 3 times the nominal stress (estimation based on stress concentration factor for a central hole in a large sheet). The risk is high that a new crack will initiate from the hole since the hole is a stress concentration and the nominal stress may be high enough to drive the crack. It does however not has to be the case since the crack may have grown out of the highest stressed volume: If the crack is large it will propagate even under a modest nominal stress.

THEORETICAL PART (14 p), cont'd

Question 4 (3p)

a)
See handout on Multi-axial fatigue. The criterion for initiation of fatigue cracks according to the Dang Van criterion can be written as: $\sigma_{EQDV} \geq \sigma_{eDV}$ where $\sigma_{EQDV} = (\sigma_{1a} - \sigma_{3a})/2 + c_{DV}\, \sigma_{h,max}$

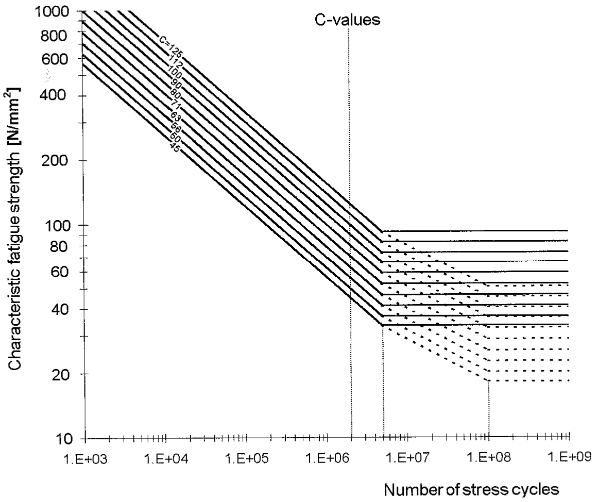
Proportional loading is assumed. Here the first term corresponds to τ_a, the maximum shear stress amplitude in a critical plane, σ_h = is the hydrostatic stress and c_{DV} and σ_{eDV} are material parameters determined from two different fatigue limits.

The equivalent stress according to Dang Van (other criteria will give the same results) will be

$$\frac{\sigma_a - 0}{2} + c_{DV}\frac{\sigma_a + \sigma_a + 0}{3} \quad \text{and} \quad \frac{\sigma_a + \sigma_a}{2} + c_{DV}\frac{\sigma_a - \sigma_a + 0}{3}$$

Thus the 180° out-of phase loading will be worst as long as $c_{DV} < 3/4$.

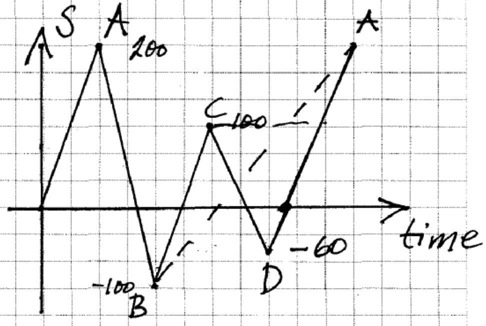
b)
For constant amplitude loading, one may have a fatigue limit at $5 \cdot 10^6$ cycles, or have a bilinear fatigue resistance curve with the fatigue resistance curve up to $n_t = 5 \cdot 10^6$ cycles and then a different slope to $n_t = 1 \cdot 10^8$ cycles where a fatigue limit is assumed, see figure below. However, for variable amplitude loading is found that there does not seem to be a fatigue limit, at least if some cycles in the load spectrum has a stress range larger than the constant amplitude fatigue limit.



PROBLEM PART (36 p)

NOTE: To obtain maximum points for each problem, the solution must be clearly motivated and all the equations used from the literature should have a clear reference (author, page and equation number)

Q5 Determine load cycles, use Dowling p 446–447



ABC no cycle

BCD no cycle

CDA yes local

$S_m = \frac{100-60}{2} = 20$

$S_a = \frac{100-(-60)}{2} = 80$

ADA yes, global

$S_m = 50$, $S_a = 150$ (MPa)

Equivalent completely reversed stress amplitude

Dowling p 488 SWT $S_{ar} = \sqrt{S_{max} \cdot S_a} \Rightarrow$

$S_{ar_l} = \sqrt{100 \cdot 80} = 89$ $\quad S_{ar_g} = \sqrt{200 \cdot 150} = 173$ MPa

Stress concentration:

$\left.\begin{aligned} \frac{d_2}{d_1} &= 1.5 \\ \frac{g}{d_1} &= 0.1 \end{aligned}\right\} \Rightarrow k_t \approx 1.85$

K_f: use Dowling p 472–474 $\rightarrow \sigma_u = 700 \rightarrow \alpha \approx 0.2 \rightarrow$

$k_f \approx 1 + q(k_t - 1) = 1 + \frac{1}{1+\frac{0.2}{1}}(1.85-1) = 1.71$

With $\sigma_{ar} = k_f \cdot S_{ar}$ and the "Basquin-relation"

$N_f = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma_f'}\right)^{1/b} \rightarrow N_{fl} = 7.1 \cdot 10^7$ $\quad N_{fg} = 1.7 \cdot 10^5$

Damage for one sequence $D = \frac{1}{N_{fl}} + \frac{1}{N_{fg}} = 5.9 \cdot 10^{-6}$

Damage $D = 1 \Rightarrow$ <u>$1.7 \cdot 10^5$</u> sequences

PROBLEM PART (36 p) cont'd

Q6 The load cycle with $\sigma_{max} = 50$ MPa will contribute to the crack growth when

$$\Delta K = (\sigma_{max} - 0) F \sqrt{\pi a_1} = \Delta K_{Th}$$

Here Dowling p 327 ⇒ $F = 1.12$ ⇒

$$50 \cdot 1.12 \sqrt{\pi a_1} = 6 \Rightarrow a_1 = 0.0037 \text{ m}$$

For the time when $a_i \leq a \leq a_1$ only the cycle with $\sigma_{max} = 150$ MPa contributes. This corresponds to N_1 cycles where we integrate

$$\frac{da}{dN} = C(\Delta K)^m \text{ , note } F \approx \text{constant (and } R = 0)$$

Dowling p 568 ⇒

$$N_1 = \frac{a_1^{1-m/2} - a_i^{1-m/2}}{C(F \Delta\sigma \cdot \sqrt{\pi})^m} \frac{1}{(1 - m/2)} = 8.25 \cdot 10^3 \text{ cycles (sequences)}$$

Check that $K_I < K_{Ic}$ when $a = a_1$ →

$$K_I(a = a_1) = \sigma_{max} F \sqrt{\pi a_1} = 150 \cdot 1.12 \sqrt{\pi \cdot 3.7 \cdot 10^{-3}} = 18 \text{ MPa}\sqrt{m}$$

$K_I(a = a_1) \leq K_{Ic} = 28$ ⇒ no static fracture before $a = a_1$

PROBLEM PART (36 p) cont'd

Q7
Use the hand-out for multi-axial fatigue, and the Crossland equivalent stress

Material parameters
Rotating bending: $400 + c_C \cdot 400/3 = \sigma_{eC}$
Pulsating bending: $340 + c_C \cdot 680/3 = \sigma_{eC}$

» $c_C = 9/14 \approx 0.64$ and $\sigma_{eC} = 3400/7 \approx 485$ MPa

Operational loading
The worst-case loading is when the stress components are in-phase. Due to the in-phase loading, the of the stress amplitude tensor will be

$$\sigma_{ij,\mathrm{a}} = \begin{bmatrix} 200 & 100 & 0 \\ 100 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with the deviatoric stress tensor

$$\sigma^{\mathrm{d}}_{ij,\mathrm{a}} = \begin{bmatrix} \frac{400}{3} & 100 & 0 \\ 100 & -\frac{200}{3} & 0 \\ 0 & 0 & -\frac{200}{3} \end{bmatrix}$$

This will give the Crossland equivalent stress as

$$\sigma_{\mathrm{eqC}} = \sqrt{\frac{3}{2}\sigma^{\mathrm{d}}_{ij,a}\sigma^{\mathrm{d}}_{ij,a}} + c_{\mathrm{C}} \cdot \sigma_{\mathrm{h,max}} = \sqrt{\frac{3}{2}\left(\left(\frac{400}{3}\right)^2 + 2\left(\frac{200}{3}\right)^2 + 2\cdot 100^2\right)} + \frac{9}{14}\cdot\frac{200}{3}$$

$$= \sqrt{\frac{3}{2}\left(\frac{400^2 + 2\cdot 200^2 + 18\cdot 100^2}{9}\right)} + \frac{300}{7} = \sqrt{70000} + \frac{300}{7} = \frac{2152}{7} \approx 307$$

This will give a safety factor of
$SF_C = 485/307 = 1.6$

2.
$SF_C = 485/\sigma_{EQC} = 1.25$ » $\sigma_{EQC} = 485/1.25 = 388$

The static loading will only contribute to the hydrostatic part. We obtain

$c_C \cdot \sigma_{xx,stat}/3 = 388 - 307 = 81$ » $\sigma_{xx,stat} = 3 \cdot 81/c_C = 3 \cdot 14 \cdot 81/9 = 378$ MPa.

3. Maximum stress

$$\sigma_{\max} = \begin{bmatrix} 200+378 & 200 & 0 \\ 200 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

gives the deviatoric stress tensor

$$\sigma^{\mathrm{d}}_{\max} = \begin{bmatrix} 2\cdot 578/3 & 200 & 0 \\ 200 & -578/3 & 0 \\ 0 & 0 & -578/3 \end{bmatrix}$$

Maximum Von Mises effective stress

$$\sigma_{\mathrm{vM}} = \sqrt{\frac{3}{2}\sigma^{\mathrm{d}}_{ij}\sigma^{\mathrm{d}}_{ij}} = \sqrt{\frac{3}{2}\left(\left(\frac{2\cdot 578}{3}\right)^2 + 2\left(-\frac{578}{3}\right)^2 + 2\cdot 200^2\right)} \approx 674$$

674 MPa < 700 MPa » no plasticity.