Fatigue Design – Assignment 2

Fatigue crack propagation and fracture

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- Supervisor: Johan Sandström

Time to finish and hand over to supervisor **2010-04-16**.

Note that *you must include all code* in your report if you are to pass the assignment.

Background

This assignment deals with crack growth. More specifically, it deals with the growth of cracks in railway rails. The crack growth is promoted by rail bending due to passing wheels. Further, in all-welded tracks (which is the common case in modern tracks) temperature deviations from the stress free temperature will induce additional (quasi-static) tensile or compressive stresses, see figure 1.

Passing wheels will induce bending moments in the rail foot. In the current tasks bending moments that result in a tensile stress in the rail foot are considered as positive, see figure 1. A measured time history is shown in figure 2. The largest positive bending moment occurs midway between two sleepers when a wheel is located in the same position. Further, there is a negative bending moment (compression in the rail foot) at this location when it is between two wheels.

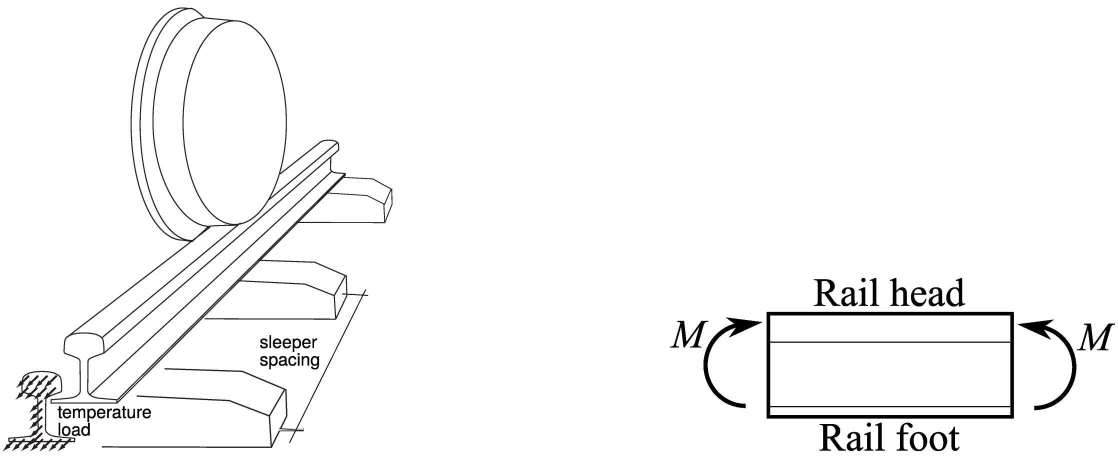


Figure 1: Wheel rolling on a rail (left). Definition of positive bending moment (right).

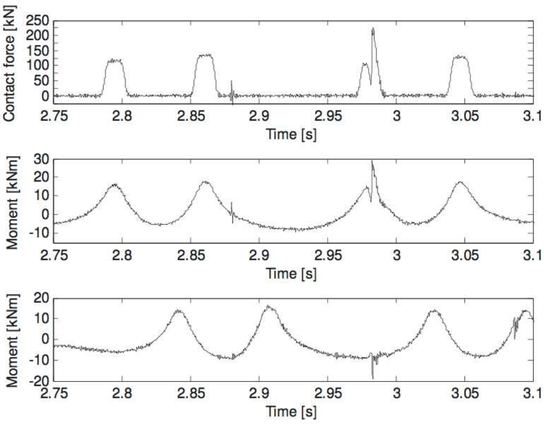
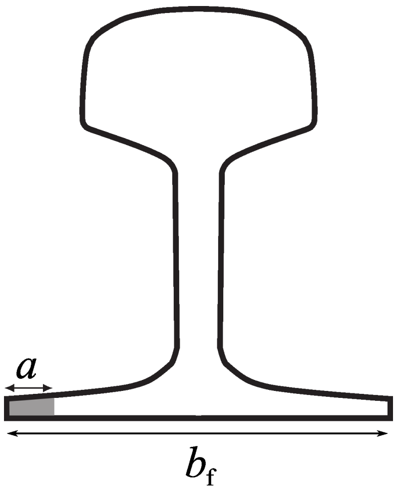


Figure 2: From top to bottom: Measured contact forces, bending moments in the middle of a sleeper span and bending moments above a sleeper.

The magnitude of the bending moment will depend on e.g. axle load, speed and support conditions. Additionally load peaks can occur due to wheel flats. An example of this is seen as the peaks in figure 2. In the following we study the case of freight waggons on the Iron Ore line between Kiruna and Luleå in the very north of Sweden. Here the axle load is 30 tonnes (as compared to all other tracks in Sweden where the maximum allowed axle load for premium lines is currently being successively upgraded from 22.5 to 25 tonnes). The sleeper distance is 60 cm.

In the current assignment we will consider an edge crack in the rail foot, see figure 3.



(a) Presumed geometry of a rail foot crack.

(b) Geometry of a rail foot crack that caused a rail break.

Figure 3: Presumed geometry of an edge crack in a rail foot.

Input data for the current assignment

The rail material has the following properties:

- Coefficient of thermal expansion $\alpha = 11.5 \cdot 10^{-6}$ [°C^{-1}]
- Elasticity modulus $E = 210$ [GPa]
- Fracture toughness $K_{\mathrm{Ic}} = 40\,[\mathrm{MPa}\sqrt{\mathrm{m}}]$
- Material parameters in Paris law $C = 2.47 \cdot 10^{-9}$ and $n = 3.3$ for $\mathrm{d}a/\mathrm{d}N$ in mm/cycle and ΔK_{I} in MPa$\sqrt{m}$.
- Yield limit $\sigma_0 = 675$ [MPa]

The rail geometry is defined by the following properties (60E1 rail profile):

- Moment of inertia $I = 30.55 \cdot 10^{-6}\,[\mathrm{m}^4]$
- Distance from neutral axis to rail foot $h_{\mathrm{f}} = 0.081$ [m]
- Rail foot width $b_{\mathrm{f}} = 0.150$ [m]
- Rail foot thickness $t_{\mathrm{f}} = 11.5$ mm

A bending moment load cycle due to a nominal (i.e. non-flatted) wheel for a crack located in the centre of the sleeper span is defined by:

- Largest positive bending moment $M_{\max} = 30$ [kNm]
- Largest negative bending moment $M_{\min} = -10$ [kNm]

Tasks

Stress analysis

1. Express the bending stress in the rail foot as a function of the applied bending moment.

2. Express the tensile/compressive stress in the rail as a function of temperature below the stress free temperature $\Delta T = T_0 - T$, where T is the current and T_0 is the stress free temperature. Note that the rail is all-welded, which means that it can be considered as clamped in the ends. Further the heating is presumed to be uniform. Does a temperature above the stress free temperature, T_0, give rise to tension or compression?

Fracture mechanics analysis

For the edge crack studied, the approximation is made that the bending gives rise to a uniform tensile (or compressive) stress in the rail foot equal to the maximum bending stress. Further, we presume that the current geometry can be approximated by the case of an edge crack in a steel sheet for which the stress intensity is

$$K_{\mathrm{I}}(a) = f(a, b_{\mathrm{f}}) \cdot \sigma\sqrt{\pi a} \tag{1}$$

with a geometry factor of

$$f(a, b_{\mathrm{f}}) = \frac{\sqrt{\frac{2b_{\mathrm{f}}}{\pi a}\tan(\frac{\pi a}{2b_{\mathrm{f}}})}}{\cos(\frac{\pi a}{2b_{\mathrm{f}}})} \cdot (0.752 + 2.02(\frac{a}{b_{\mathrm{f}}}) + 0.37(1 - \sin(\frac{\pi a}{2b_{\mathrm{f}}}))^3) \tag{2}$$

3. For the given maximum bending moment, evaluate critical crack sizes for temperatures for $\Delta T = 0\,°\mathrm{C}$, $\Delta T = 10\,°\mathrm{C}$ and $\Delta T = 40\,°\mathrm{C}$.

Note that the stress free temperature on the Iron Ore line is around 10 °C. Thus, $\Delta T = 40\,°\mathrm{C}$ corresponds to a temperature of $T = -30\,°\mathrm{C}$, which occurs regularly.

Crack growth analysis

4. Transform the given material parameters C and n so that they correspond to $\mathrm{d}a/\mathrm{d}N$ in m/cycle and ΔK_{I} in $\mathrm{Pa}\sqrt{m}$.

5. Evaluate the growth of an edge crack in the rail foot from an initial size of $a_{\mathrm{ini}} = 0.5$ mm until fracture. Plot crack size as function of number of load cycles for the temperatures $\Delta T = 0\,°\mathrm{C}$, $\Delta T = 10\,°\mathrm{C}$ and $\Delta T = 40\,°\mathrm{C}$.. Compare with an evaluation of growth from an initial size of $a_{\mathrm{ini}} = 5$ mm. Stress ratio effects need not be accounted for. However, you are obliged to state whether accounting for them will result in a longer or shorter fatigue life.

6. Consider (no requirement to write) which of the two a_{ini} magnitudes in task 5 that may be most relevant.
Hint: There is really no clear-cut answer, but consider cases where each may be relevant.

7. Evaluate for which crack lengths LEFM is valid under static and cyclic loading. Which limit magnitudes (static or cyclic loading) is relevant for our cases of crack growth and fracture? Discuss the consequences on your predicted life times and limit loads.

APPENDIX
Numerical evaluation of crack growth

1. Read material parameters E, α, C, n, K_{Ic}.

2. Read geometric data for the rail profile I, h_f, b_f.

3. Evaluate thermal stress.

4. Evaluate maximum and minimum bending stresses.

5. Evaluate crack growth. Probably the easiest way is to take advantage of the (presumed) constant amplitude loading and adopt a numerical integration of Paris law as something like
   `Qv = quad('dK_foot',a_start,a_end(j));`
   Here `dK_foot` is a function that evaluates stress intensity ranges, `a_start` is the initial size of the crack and `a_end(j)` the final length of the crack. If you like you can compare to a "brute-force" cycle by cycle integration.
   Some things to consider:

   a) In evaluating the range of the stress intensity factor, ΔK, note that the thermal stress is constant. Also note that K_{min} is never negative!

   b) You will need the stresses in the function `dK_foot`. Two ways of achieving this is either to hard-code the stress evaluation (with indata) in `dK_foot` or use global variables (see the MATLAB help files).

   c) Since you are required to plot crack size as function of the number of load cycles you will need to perform the numerical integration not only for the final (critical) crack size, but also for intermediate crack sizes. in the example these have been stored in the vector `a_end`.

   d) Consider how to handle the material parameters C, n. To not confuse units, it may be wise to adopt values corresponding to $\mathrm{d}a/\mathrm{d}N$ in m/cycle and ΔK_{I} in $\mathrm{Pa}\sqrt{\mathrm{m}}$. However, be aware that that this may lead to numerical problems, in particular in the case when the initial crack size is 0.5 mm if C and n are extracted from the actual integration procedure.

6. Establish critical crack size. A simple way to do this is to calculate the maximum stress intensity factor for every crack size considered in the crack growth analysis (to make it simpler, it could be wise to do this separately after the crack growth evaluation using a different function). Then go through the vector with maximum stress intensity factors and check for $K_{\mathrm{I,max}} > K_{\mathrm{Ic}}$. (You can either do this backwards through the vector or adopt a break condition if the inequality is fulfilled.)