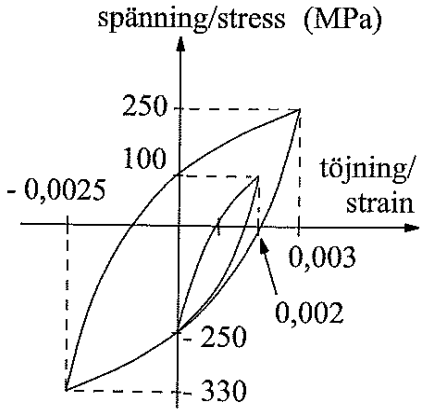
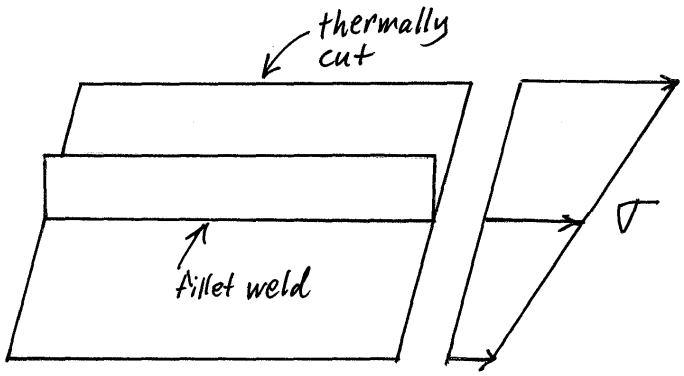
Fatigue Design (MMA115) Problem solving 2010-05-17

The diagram shows a stabilized hysteresis loop for a material subjected to a load sequence of two cycles. The two cycles give a stress-strain curve according to the Figure. Use the modified Morrow relationship to determine the number of cycles to fatigue failure. Use the material data: E = 200 GPa, σ_f' = 900 MPa, ε_f' = 0.26, b = -0.095, c = -0.47.



The Figure below shows a stiffened plate loaded by a longitudinal membrane stress that varies linearly between the lower and upper edges. The stiffener is fillet welded to the plate using mechanized welding with the weld quality WC. The upper edge of the plate is thermally cut with the surface roughness < 0.3 mm. The stress range at the upper edge is σ_r =200 MPa and the stress range at the weld position is σ_r =180 MPa.
Estimate the number of cycles to failure n_t if the risk for failure is Q_B = 0.1. The longitudinal membrane stress has the stress ratio R = -1 and the spectrum parameter κ = 1/3. The plate thickness is t = 10 mm. You may put γ_f = 1.

Can the number of cycles to failure n_t be increased if the weld, but not the gas cut surface, is shot peened so that the longitudinal welding residual stress is reduced to σ_{res} = - 100 MPa?



PROBLEM PART (36 p)

NOTE: To obtain maximum points for each problem, the solutions must be clearly motivated and all the equations used from the literature should have a clear reference (author, page and equation number).

Question 5 (12 p)

A flat bar with a rectangular cross-section $t \times b$ is cyclically loaded in the axial direction by a zero to tension load with maximum $P_{\max}$. The endurance limit of the material is σ_{e} and the ultimate tensile strength is σ_{u}.

What is the largest diameter of a centrally located hole (see Figure A8 in Dowling's book on p.789) that can be allowed for if a safety factor of 2 against fatigue crack initiation should be fulfilled?

For size, surface and mode of load corrections, a factor of $m = 0.85$ may be used. It is recommended to use the Smith-Watson-Topper (SWT) approach for mean value influence, and the Peterson formula for notch sensitivity.

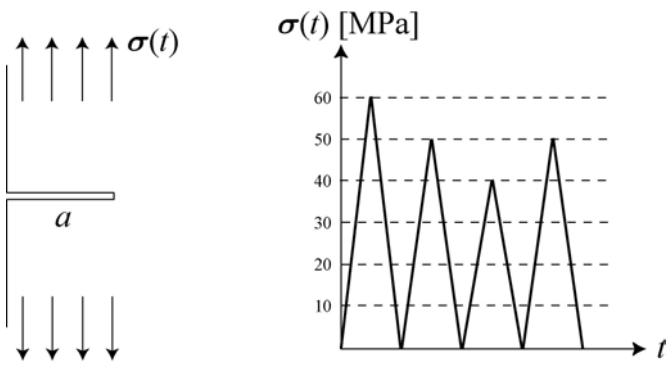
Input data: $t = 10$ mm; $b = 40$ mm; $P_{\max} = 24$ kN; $\sigma_{\mathrm{e}} = 300$ MPa; $\sigma_{\mathrm{u}} = 640$ MPa.

Question 6 (12 p)

A wide sheet in an aircraft structure loaded by a uniform stress $\sigma(t)$ may develop edge cracks according to the figure below. The stress varies according to the stress history below and then this sequence is repeated.

How many sequences can be allowed between inspections?

Cracks larger than $a = 2$ mm can be detected and the maximum stress-intensity factor allowed is $K_{\mathrm{Ic}} = 30\ \mathrm{MPa}\sqrt{\mathrm{m}}$. The yield stress is $\sigma_{\mathrm{y}} = 600$ MPa, and the threshold value for crack growth is $K_{\mathrm{th}} = 2\ \mathrm{MPa}\sqrt{\mathrm{m}}$ at $R = 0$. For the material, the relationship $\mathrm{d}a/\mathrm{d}N = 10^{-11}\left(\Delta K\right)^4$ [m/cycle] may be used with ΔK in units of $\mathrm{MPa}\sqrt{\mathrm{m}}$.



Question 7 (12 p)

A shaft is subjected to reversed torsion and bending. The resulting stresses are $\tau_{x\varphi} = \pm 100$ MPa and $\sigma_x = \pm 300$ MPa, respectively. Presume these stress components to be proportional in time.
The material has fatigue limits in reversed torsion and in rotating bending of $\tau_{\mathrm{FL}} = 250$ MPa and $\sigma_{\mathrm{FLB}} = 400$ MPa, respectively. All corrections for surface roughness etc are presumed to be included in these fatigue limits.

a) Evaluate the safety factor against fatigue initiation according to the Dang Van criterion!
b) Since this safety factor was deemed as not acceptable, the shaft was subjected to a surface treatment that introduced a compressive residual stress at the surface of $\sigma_x = \sigma_\varphi = -150$ MPa. Evaluate which safety factor against fatigue initiation according to Dang Van this would result in!