Chalmers University of Technology
Department of Applied Mechanics

Brief solutions to exam in Artificial Intelligence 2, CIU036, 2009-01-27, 09.00-13.00, Vin Cerf.
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1. (a) See page 51 in the course book.
   (b) See page 54 in the course book.
   (c) See page 125 in the course book.
   (d) See page 73 in the course book.
   (e) See page 36 in the course book.

2. (a) Description of AS, see pp. 105–107 in the course book. For full points, the description should contain all the steps (1-4), as well as a clear explanation of (1) pheromone initialization, (2) probabilistic path generation, and (3) the rules for updating pheromones.
   (b) The main differences between MMAS and AS are that
   - In MMAS, only the ant generating the best solution is allowed to deposit pheromone. The definition of the best solution is typically changed during a run, so that one uses *best so far* for some iterations, then *best in current generation* for some iterations etc.
   - In MMAS, one introduces limits on the pheromone levels. Thus, if the pheromone level τ_{ij} on a given edge e_{ij} falls below $\tau_{\min}$, it is set to $\tau_{\min}$. Similarly, if the pheromone level τ_{ij} exceeds $\tau_{\max}$, it is set to $\tau_{\max}$.
   - In MMAS, pheromones are initialized to the maximum level, i.e. such that

   $$\tau_{ij} = \tau_{\max} \forall (i, j) \in \{1, n\}$$

   $\tau_{\max}$ is set as $1/(\rho D_b)$, where ρ is the evaporation rate and D_b is the length of the current best tour.

3. (a) i. Using roulette-wheel selection, the probability of selecting individual 4 can be written as

   $$p_4 = \frac{F_4}{F_1 + F_2 + F_3 + F_4 + F_5} = \frac{16}{55} \approx 0.291$$

   ii. In the case of tournament selection with tournament size 2, there are $5 \times 5 = 25$ possible tournaments, since the individuals are chosen (for the tournament) with replacement. Thus, the possible pairs of individuals are

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(1,1), (1,2), ... (5,5). Of these 25 pairs (which occur with equal probability, namely 1/25), 9 involve individual 4: (1,4), (2,4), (3,4), (4,4), (4,5), (4,1), (4,2), (4,3), (5,4). For six of the pairs individual 4 is the better individual (and is thus selected with probability p_{tour}) whereas for two of the pairs ((4,5) and (5,4)) the other individual is better, so that individual 4 is selected only with probability $1 - p_{\text{tour}}$. For the pair (4,4), individual 4 is obviously selected with probability 1. Thus, summarizing, the probability of selecting individual 4 equals

$$\frac{1}{25}\left(6p_{\text{tour}} + 2(1 - p_{\text{tour}}) + 1\right) = 0.24$$

(b) When decoded, the six chromosomes give the following variable values: $x_1 = 0.625$, $y_1 = 0.8125$, $x_2 = 0.375$, $y_2 = 0.4375$, $x_3 = 0.4375$, $y_3 = 0.3125$, $x_4 = 0.3125$, $y_4 = 0.5625$, $x_5 = 0.5625$, $y_5 = 0.0625$, $x_6 = 0.5000$, $y_6 = 0.5625$, from which the fitness values can be computed as $f_i = x_i^2 + y_i^2$. The average fitness $\overline{f}$ equals 0.49544, and the average fitness of S_1 equals $\overline{f}(S_1) = 0.80859375$. Inserting these numbers, and the parameters given in the problem formulation, in the schema theorem, the resulting expected number of copies of S_1 becomes $3.01234 \approx 3$.

4. (a) Description of basic PSO algorithm, see pp. 120–124 in the course book (Algorithm 5.1).

   (b) In PSO, the tradeoff between exploration and exploitation is handled using the inertia weight w. The velocity change according to

$$v_{i,j} \leftarrow w v_{i,j} + c_1 q\left(\frac{x_{i,j}^{\text{pb}} - x_{i,j}}{\Delta t}\right) + c_2 r\left(\frac{x_j^{\text{sb}} - x_{i,j}}{\Delta t}\right),\ i = 1,\dots,N,\ j = 1,\dots,n$$

   where xij denotes position component j of particle i, v_{ij} denotes velocity component j of particle i, c_1 and c_2 are constants, $x_{i,j}^{\text{pb}}$ are the components of the best position found by particle i and x_j^{sb} are the components of the best position found by any particle in the swarm. If $w > 1$, the search puts more emphasis on exploration, since the cognitive and social component (the terms involving c_1 and c_2) then play a less significant role than if $w < 1$, in which case the PSO algorithm tries to exploit the results already found, as encoded in the cognitive and social components. Initially, w is typically set to a value larger than 1 (1.4, say), and is then lowered down to a limit of around 0.3-0.4. A common procedure for reducing w is through multiplication by a factor $\beta \in]0, 1]$ (often very close to 1).

5. See pp. 63–65 (Example 3.8) in the course book.

   (a) See Eqn. 3.42 in the book for the probability distribution for generation 3.

   (b) See Eqn. 3.45 in the book for the probability distribution for generation 4.